value than the short put and, thus, our long position loses faster than our short position. As time goes by, we lose from this effect. At low stock prices, the short put has more time value than our long put and, thus, our short position loses faster than our long position. As time goes by, we gain from this effect.

We should repeat, however, that these statements do not advocate a short or long holding period, because the length of the holding period affects the range of possible stock prices.

A Note about Call Bear Spreads and Put Bull Spreads

As we have seen, it is possible to design a call money spread that will profit in a bull market. It is also possible to construct a call money spread that will profit in a bear market. There is, however, a risk of early exercise. The short call can be sufficiently in-the-money to justify early exercise, while the long call is still out-of-the-money. Even if the long call is in-the-money, the cash flow from early exercise will be negative.

For example, suppose that S_t is the stock price prior to expiration. The cash flow from the exercise of the short call is $-(S_t - X_1)$, while the cash flow from the exercise of the long call is $S_t - X_2$. This gives a total cash flow of $X_1 - X_2$, which is negative. Early exercise ensures that the bear spreader will incur a cash outflow. Because the loss occurs prior to expiration, it is greater in present value terms than if it had occurred at expiration. Thus, the call bear spread entails a risk not associated with the bull spread.

Just as we can construct bear money spreads with calls, we can also construct bull money spreads with puts. Here we would buy the low exercise price put and sell the high exercise price put. The pattern of payoffs would be similar to those of the call bull money spread, but, as with call bear money spreads, early exercise would pose a risk.

Collars

Now we shall look at a popular strategy often used by professional money managers and referred to as a collar. A collar is very similar to a bull spread. In fact, the relationship between the two can be seen by applying what we learned about put-call parity.

Suppose that you buy a stock. You wish to protect it against a loss but participate in any gains. An obvious strategy is one we covered in Chapter 6, the protective put. If you buy the put, you will have to pay out cash for the price of the put. The collar reduces the cost of the put by adding a short position in a call, where the exercise price is higher than the exercise price of the put. Although a call with any exercise price can be chosen, there is in fact one particular call that tends to be preferred: the one whose price is the same as that of the put you are buying.

Thus, let us buy the stock and buy a put with an exercise price of X_1 at a price of P_1. Now let us sell a call at an exercise price of X_2 with a premium of C_2. With $N_P = 1$ and $N_C = -1$ the profit equation is

$$\Pi = S_T - S_0 + \text{Max}(0, X_1 - S_T) - P_1 - \text{Max}(0, S_T - X_2) + C_2.$$

The profits for the three ranges are as follows:

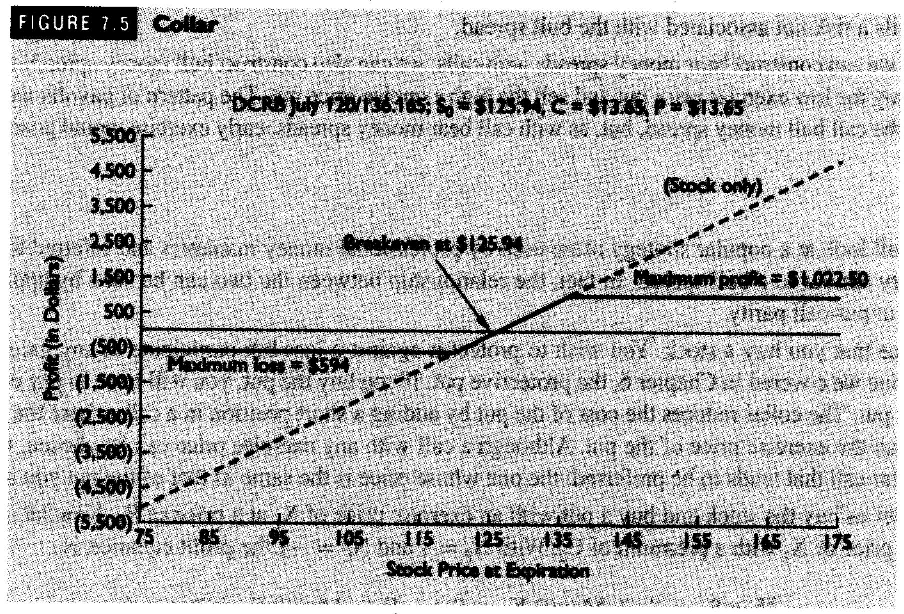
$$\begin{aligned}
\Pi &= S_T - S_0 + X_1 - S_T - P_1 + C_2 && \text{if } S_T \le X_1 < X_2.\\
&= X_1 - S_0 - P_1 + C_2 &&\\
\Pi &= S_T - S_0 - P_1 + C_2 && \text{if } X_1 < S_T < X_2.\\
\Pi &= S_T - S_0 - P_1 - S_T + X_2 + C_2 && \text{if } X_1 < X_2 \le S_T.\\
&= X_2 - S_0 - P_1 + C_2 &&
\end{aligned}$$

Since X_1 is below the current stock price, S_0, and X_2 is above it, the profit on the stock is either $X_1 - S_0$, which is negative, when S_T is at X_1 or below, or $X_2 - S_0$ which is positive, when S_T is at X_2 or above. Thus, the potential loss and gain on the stock are fixed and limited. Only in the middle range, where S_T is essentially between the two exercise prices, is there any uncertainty. As noted above, it is common to set X_2 such that $C_2 = P_1$, so these terms drop out of the above profit equations. It is not necessary, however, that we choose the call such that its price offsets the price paid for the put. In such a case, we must add $-P_1 + C_2$ to the stock profit, as indicated in the above equations.

When the call and put premiums do offset each other, the collar is sometimes referred to as a zero-cost collar, but this term is somewhat misleading. While there is no cash outlay for the options, the cost is in the willingness to give up all gains beyond X_2. In other words, the investor will be selling the stock at a maximum price of X_2, in exchange for which the investor receives the assurance that the stock will be sold for no worse than X_1.

Although collars are normally used with index options in conjunction with diversified portfolios, we shall stay with the example here of the single stock, DCRB. Figure 7.5 illustrates the collar for the DCRB July 120/136.165. The stock is bought at 125.94. Let us say we buy the put with an exercise price of 120, which costs 13.65. Now, we need to sell a call with an exercise price such that its premium is 13.65. For the July options we see that none of the calls have a price of 13.65. The 130 has a price of 16.40, so we will need a call with an exercise price greater than \$130. We can use the Black-Scholes-Merton model to figure out what the exercise price should be. We use the following inputs: $S_0 = 125.94$, $X = 136.165$, $r_c = 0.0453$, $\sigma = 0.83$, and $T = 0.1726$. We find that this call has a Black-Scholes-Merton price of 13.65.

FIGURE 7.5 **Collar**



Now we are faced with a problem we have not yet seen. If we use exchange-listed options, we cannot normally designate the exercise price, for this is set by the exchange. We could sell the 135 or the 140, but there is no 136.165. We have two choices. We can use an over-the-counter option, as discussed in Chapter 2, which can be customized to any exercise price. We go to an options dealer and request this specific option. Alternately, if we trade in sufficient volume, we can use FLEX options, also mentioned in Chapter 2, which trade on the exchanges and permit us to set the exercise price. Let us assume that we do one or the other. For illustrative purposes it does not matter.

Thus, we buy the 120 put for \$13.65 and sell the 136.165 call for \$13.65. We do 100 of each and buy 100 shares of the DCRB stock. Note that the maximum profit is capped at $100(X_2 - S_0)$, $= 100(136.165 - 125.94) = 1{,}022.50$, and the maximum loss is $100(S_0 - X_1) = 100(125.94 - 120) = 594$. The breakeven is found by setting the middle equation to zero:

$$\Pi = S_T^* - S_0 = 0,$$

which we see is obviously

$$S_T^* = S_0,$$

and in our problem, $S_T^* = 125.94$, the original stock price. Note in Figure 7.5 that the line for the stock profit passes right through the middle range of profit for the collar. So, the investor effectively buys the stock and establishes a maximum loss of \$594 and a maximum gain of \$1,022.50. Note that the maximum profit is earned with an upward move of about 8.1 percent. The maximum loss is incurred with a downward move of about 4.7 percent.

This strategy looks a lot like a bull spread. Let us examine the difference. For the three ranges of S_T, the profit equations for the bull spread and collar are given below. For the collar, we substitute from put-call parity, $C_1 - S_0 + X_1(1 + r)^{-T}$ for P_1.

$S_T \leq X_1 < X_2$.

Bull spread:	$-C_1 + C_2$.
Collar:	$X_1 - S_0 - P_1 + C_2$
	$= X_1 - X_1(1 + r)^{-T} - C_1 + C_2$.

$X_1 < S_T < X_2$.

Bull spread:	$S_T - X_1 - C_1 + C_2$.
Collar:	$S_T - S_0 - P_1 + C_2$
	$= S_T - X_1(1 + r)^{-T} - C_1 + C_2$.

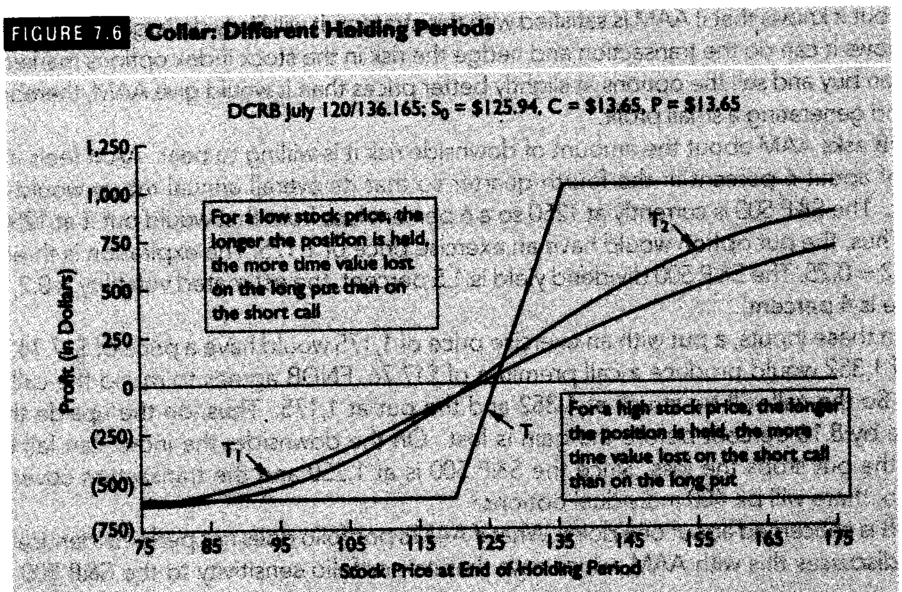
$X_1 < X_2 \leq S_T$.

Bull spread:	$X_2 - X_1 - C_1 + C_2$.
Collar:	$X_2 - S_0 - P_1 + C_2$
	$= X_2 - X_1(1 + r)^{-T} - C_1 + C_2$.

Thus, in all cases the collar is more profitable by the difference between X_1 and the present value of X_1.

Thus, the bull spread and the collar are similar but not identical. Note that the range of profits for the bull spread is $X_2 - X_1 - C_1 + C_2 - (-C_1 + C_2) = X_2 - X_1$. The range of profits for the collar is $X_2 - X_1(1 + r)^{-T} - C_1 + C_2 - (X_1 - X_1(1 + r)^{-T} - C_1 + C_2) = X_2 - X_1$. So the two strategies have the same range of profits. The initial outlay for the bull spread is $C_1 - C_2$. The initial outlay for the collar is $S_0 + P_1 - C_2$. Substituting from

FIGURE 7.6 Collar: Different Holding Periods



put-call parity for P_1, this is $X_1(1 + r)^{-T} + C_1 - C_2$. The collar is equivalent to a bull spread plus a risk-free bond paying X_1 at expiration.

Choice of Holding Period In Figure 7.6 we see what the profit would look like if the position were closed early. On the downside, by closing the position at T_1, we recoup more of the time value on the long put than we pay to buy back the short call. The longer we hold the position on the downside, the less this works to our advantage. On the upside, if we close the position early, the more time value we must buy back on the short call than we receive from selling the long put. The longer we hold the position, the more this works to our advantage. Remember that these statements are true because the time value is greater where the stock price is close to the exercise price. On the downside, the time value is greater on the long put; on the upside, the time value is greater on the short call.

Derivatives Tools

Concepts, Applications, and Extensions

Designing a Collar for an Investment Portfolio

Avalon Asset Management (AAM) is a (fictional) small investment management company with $50 million of assets under management. Its performance is measured at the end of the calendar year. Through nine months this year, AAM has earned outstanding returns for its clients, with its overall portfolio up about 21 percent. AAM is, however, concerned about the fourth quarter. It has worked hard for its performance year to date and would not want to see it evaporate.

A partner has learned of the collar strategy, which would enable it to purchase insurance against downside losses, in the form of puts, by selling off some of its upside gains, in the form of calls. AAM has never used options before but feels that it understands the risks and rewards. It is not authorized, however, to use options for any of its client accounts. The partners decide to experiment with the collar strategy using $500,000 from the company's pension fund. The partners know that the collar is a conservative strategy and will not jeopardize the pension fund.

The partners approach First National Dealer Bank (FNDB) with a request to purchase a collar covering $500,000 of the portfolio. FNDB knows that this is a small derivatives transaction, which it would ordinarily not do, but it knows that if AAM is satisfied with this strategy, it will likely do larger transactions later. The bank knows it can do the transaction and hedge the risk in the stock index options market. It also feels that it can buy and sell the options at slightly better prices than it would give AAM, thereby covering its costs and generating a small profit.

FNDB asks AAM about the amount of downside risk it is willing to bear. AAM feels it can tolerate a loss of about 6 percent in the fourth quarter so that its overall annual return would be about 15 percent. The S&P 500 is currently at 1250 so a 6 percent loss from that would put it at $1250(1 - 0.06) = 1{,}175$. Thus, the put option would have an exercise price of 1,175. The expiration is three months, so $T = 3/12 = 0.25$. The S&P 500 dividend yield is 1.5 percent, the estimated volatility is 0.2, and the risk-free rate is 4 percent.

Given these inputs, a put with an exercise price of 1,175 would have a price of $17.74. An exercise price of 1,352 would produce a call premium of $17.76. FNDB agrees to round the call premium to $17.74. So the call will be struck at 1,352 and the put at 1,175. Thus, on the upside the index can increase by 8.16 percent before the gain is lost. On the downside, the index can fall by 6 percent before the put stops the loss. Since the S&P 500 is at 1,250 and the transaction covers a $500,000 portfolio, there will be 400 individual options.

FNDB is concerned about one point. What if AAM's portfolio does not perform identically to the S&P 500? It discusses this with AAM, which says that its portfolio sensitivity to the S&P 500 is about 101 percent. In technical terms, the beta is 1.01. AAM is satisfied that this is a close enough match to the S&P 500. FNDB is not so sure, but feels that the risk is worth taking. So the collar is executed.

During the final three months of the year, the market surprisingly continues to perform well. The portfolio rises 7.5 percent to \$537,500. The S&P 500, however, outperforms the portfolio, increasing at a 10 percent rate to 1375. The call options expire in-the-money and AAM must pay 400Max(0,1375 – 1352) = \$9,200. This effectively reduces the value of the overall position to \$537,500 – \$9,200 = \$528,300. The rate of return is, therefore,

$$\frac{\$528{,}300}{\$500{,}000} - 1 = 0.0566$$

This is an overall return of a little over three-fourths of what the portfolio earned. The partners are confused. They believed they had room on the upside to earn the full return up to about 8 percent. With their portfolio slightly more volatile than the market, they believed that if they lost anything, it would be the excess of the portfolio's performance relative to the S&P 500. So what happened?

The portfolio underperformed the S&P 500. Even though the portfolio was thought to be more volatile than the S&P 500, measuring a portfolio's volatility is difficult. With the portfolio underperforming the S&P 500, the short call expired in-the-money without a corresponding gain on the portfolio to offset. Had the portfolio grown by 10 percent, the performance of the S&P 500, its value would have been \$550,000. Deducting the \$9,200 payoff on the call, the total value would have been \$550,000 – \$9,200 = \$540,800, a gain of 8.16 percent, which is the precise upside margin built into the collar.

AAM attempted to protect a portfolio using options in which the underlying was not identical to the portfolio. Whether it uses the collar strategy again will depend on its tolerance for small discrepancies in performance from its target. AAM is, however, generally pleased because its annual performance was enhanced through its fourth quarter performance, though not as much as its performance would otherwise have been.

Butterfly Spreads

A butterfly spread, sometimes called a sandwich spread, is a combination of a bull spread and a bear spread. However, this transaction involves three exercise prices: X_1, X_2, and X_3, where X_2 is halfway between X_1 and X_3. Suppose we construct a call bull spread by purchasing the call with the low exercise price, X_1, and writing the call with the middle exercise price, X_2. Then we also construct a call bear spread by purchasing the call with the high exercise price, X_3, and writing the call with the middle exercise price, X_2. Combining these positions shows that we are long one each of the low- and high-exercise-price options and short two middle-exercise-price options. Since $N_1 = 1$, $N_2 = -2$, and $N_3 = 1$, the profit equation is

$$\Pi = \text{Max}(0, S_T - X_1) - C_1 - 2\text{Max}(0, S_T - X_2) + 2C_2 + \text{Max}(0, S_T - X_3) - C_3.$$

To analyze the behavior of the profit equation, we must examine four ranges of the stock price at expiration:

$$\Pi = -C_1 + 2C_2 - C_3 \qquad \text{if } S_T \le X_1 < X_2 < X_3.$$

$$\Pi = S_T - X_1 - C_1 + 2C_2 - C_3 \qquad \text{if } X_1 < S_T \le X_2 < X_3.$$

$$\begin{aligned}\Pi &= S_T - X_1 - C_1 - 2S_T + 2X_2 + 2C_2 - C_3 \\ &= -S_T + 2X_2 - X_1 - C_1 + 2C_2 - C_3 \qquad \text{if } X_1 < X_2 < S_T \le X_3.\end{aligned}$$

$$\begin{aligned}\Pi &= S_T - X_1 - C_1 - 2S_T + 2X_2 + 2C_2 + S_T - X_3 - C_3 \\ &= -X_1 + 2X_2 - X_3 - C_1 + 2C_2 - C_3 \qquad \text{if } X_1 < X_2 < X_3 < S_T.\end{aligned}$$

Now, look at the first profit equation, $-C_1 + 2C_2 - C_3$. This can be separated into $-C_1 + C_2$ and $C_2 - C_3$. We already know that a low-exercise-price call is worth more than a high-exercise-price call. Thus, the first pair of terms is negative and the second pair is positive. Which pair will be greater in an absolute sense? The first pair will. The advantage of a low-exercise-price call over a high-exercise-price call is smaller at higher exercise prices, because there the likelihood of both calls expiring out-of-the-money is greater. If that happens, neither call will be of any value to the trader. Because $-C_1 + C_2$ is larger in an absolute sense than $C_2 - C_3$ the profit for the lowest range of stock prices at expiration is negative.

For the second range, the profit is $S_T - X_1 - C_1 + 2C_2 - C_3$. The last three terms, $-C_1 + 2C_2 - C_3$, represent the net price paid for the butterfly spread. Because the stock price at expiration has a direct effect on the profit, a graph would show the profit varying dollar for dollar and in a positive manner with the stock price at expiration. The profit in this range of stock prices can, however, be either positive or negative. This implies that there is a breakeven stock price at expiration. To find that stock price, S_T^* set this profit equal to zero:

$$S_T^* - X_1 - C_1 + 2C_2 - C_3 = 0.$$

Solving for S_T^* gives

$$S_T^* = X_1 + C_1 + 2C_2 + C_3.$$

The breakeven equation indicates that a butterfly spread is profitable if the stock price at expiration exceeds the low exercise price by an amount large enough to cover the net price paid for the spread.

Now look at the third profit equation. Since the profit varies inversely dollar for dollar with the stock price at expiration, a graph would show the profit decreasing one for one with the stock price at expiration. The profit can be either positive or negative; hence, there is a second breakeven stock price. To find it, set the profit equal to zero:

$$S_T^* + 2X_2 - X_1 - C_1 + 2C_2 - C_3 = 0.$$

Solving for S_T^* gives

$$S_T^* = 2X_2 - X_1 - C_1 + 2C_2 - C_3$$

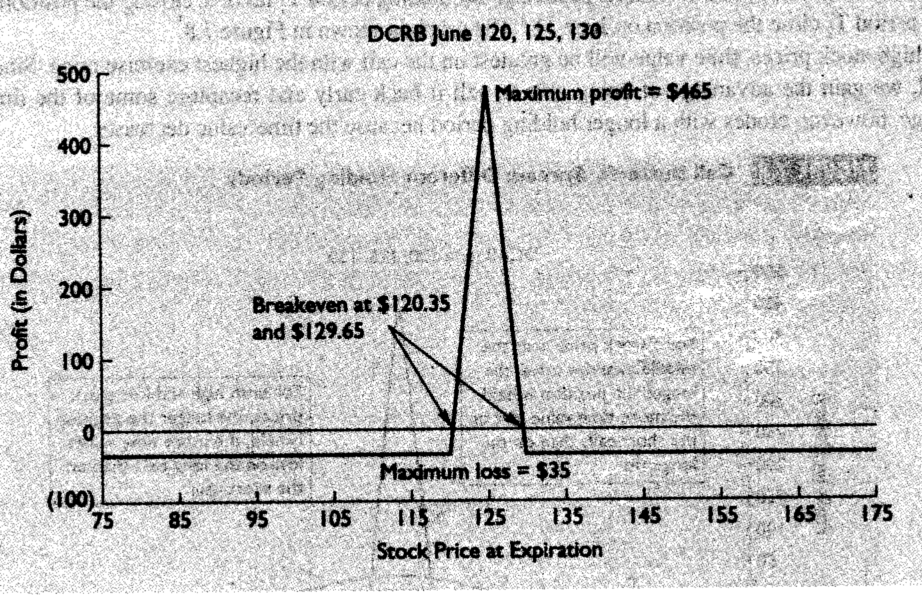
Recall that in this range of stock prices, the profit declines with higher stock prices. Profit will disappear completely if the stock price is so high that it exceeds the cash flow received from the exercise of the middle-exercise-price call, $2X_2$, minus the cash flow paid for the exercise of the low-exercise-price call, X_1, minus the net premiums on the calls.

In the final range of the stock price at expiration, profit is the net premiums paid plus the difference in the exercise prices. Since X_2 is halfway between X_1 and X_3, then $X_2 - X_1$ is the same as $X_3 - X_2$. Therefore, $-X_1 + 2X_2 - X_3 = 0$. This means that the profit in this range is the same as that in the first range and is simply the difference in the premiums.

Now that we have a good idea of what a butterfly spread looks like, consider the DCRB June 120, 125, and 130 calls. In this example, a plot of the results would reveal that the butterfly spread would profit at any stock price. Upon further inspection of the prices, we would see that the cost of buying the butterfly spread is less than the lowest possible value of the spread at expiration. Therefore, one or more of the options must be mispriced. To avoid any confusion about the performance of the butterfly spread, we should use theoretically correct prices, which can be obtained from the Black-Scholes-Merton model. Using the volatility of 83 percent, we would see that at a market price of \$15.40, the 120 call is significantly lower than its Black-Scholes-Merton value of about \$16. Thus, let us use \$16 as its price.

Figure 7.7 illustrates the butterfly spread for the June 120, 125, and 130 calls with premiums of \$16.00, \$13.50, and \$11.35, respectively. The worst outcome is simply the net premiums or $100[-16.00 + 2(13.50) - 11.35] = -35$. This is obtained for any stock price less than \$120 or greater than \$130. The maximum profit is

FIGURE 7.7 Call Butterfly Spread



obtained when the stock price at expiration is at the middle exercise price. Using the second profit equation and letting $S_T = X_2$, the maximum profit is

$$\Pi = X_2 - X_1 - C_1 + 2C_2 - C_3,$$

which in this example is 100[125 − 120 − 16.00 + 2(13.50) − 11.35] = 465. The lower breakeven is $X_1 + C_1 - 2C_2 + C_3$, which in this case is 120 + 16.00 − 2(13.50) + 11.35 = 120.35. The upper breakeven is $2X_2 - X_1 - C_1 + 2C_2 - C_3$, which in this example is 2(125) − 120 − 16.00 + 2(13.50) − 11.35 = 129.65.

The butterfly spread strategy assumes that the stock price will fluctuate very little. In this example, the trader is betting that the stock price will stay within the range of $120.35, a downward move of 4.4 percent to $129.65, an upward move of 2.9 percent. If this prediction of low stock price volatility proves incorrect, however, the potential loss will be limited—in this case, to $35. Thus, the butterfly spread is a low-risk transaction.

A trader who believes the stock price will be extremely volatile and will fall outside of the two breakeven stock prices might want to write a butterfly spread. This will involve one short position in each of the X_1 and X_3 calls and two long positions in the X_2 call. We shall leave it as an end-of-chapter problem to explore the short butterfly spread.

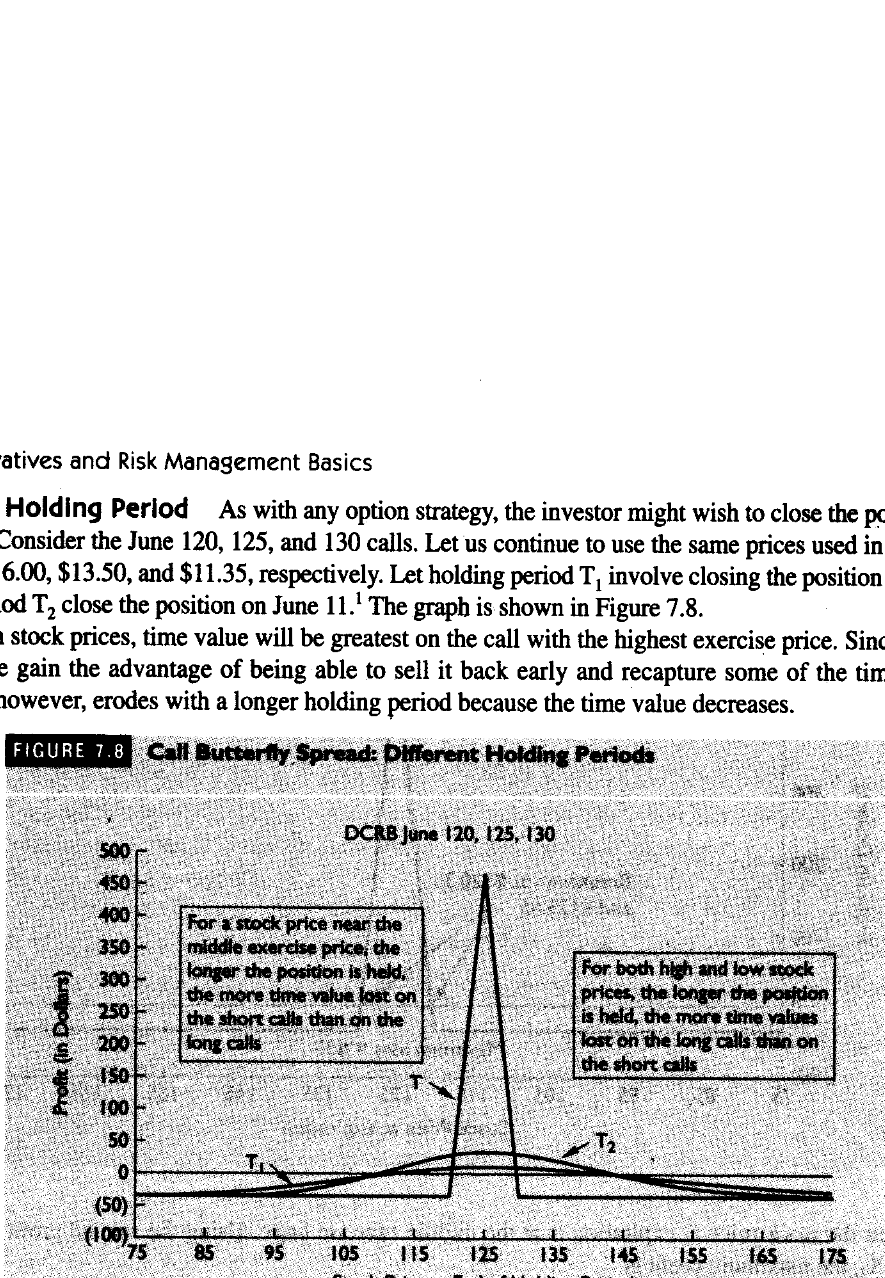
Early exercise can pose a problem for holders of butterfly spreads. Suppose the stock price prior to expiration is S_t, where S_t is greater than or equal to X_2 and less than or equal to X_3. Assume that the short calls are exercised shortly before the stock goes ex-dividend. The spreader then exercises the long call with exercise price X_1. The cash flow from the short calls is $-(2S_t - 2X_2)$, and the cash flow from the long call is $S_t - X_1$. This gives a total cash flow of $-S_t + X_2 + X_2 - X_1$. The minimum value of this expression is $-X_2 + X_2 + X_2 - X_1 = X_2 - X_1$, which is positive. If S_t exceeds X_3 and the two short calls are exercised, they will be offset by the exercise of both long calls, and the overall cash flow will be zero.

Thus, early exercise does not result in a cash outflow, but that does not mean that it poses no risk. If the options are exercised early, there is no possibility of achieving the maximum profit obtainable at expiration when $S_T = X_2$. If the spread were reversed and the X_1 and X_3 calls were sold while two of the X_2 calls were bought, early exercise could generate a negative cash flow.

Choice of Holding Period As with any option strategy, the investor might wish to close the position prior to expiration. Consider the June 120, 125, and 130 calls. Let us continue to use the same prices used in the preceding example, \$16.00, \$13.50, and \$11.35, respectively. Let holding period T_1 involve closing the position on June 1 and holding period T_2 close the position on June 11.[1] The graph is shown in Figure 7.8.

At high stock prices, time value will be greatest on the call with the highest exercise price. Since we are long that call, we gain the advantage of being able to sell it back early and recapture some of the time value. This advantage, however, erodes with a longer holding period because the time value decreases.

FIGURE 7.8 Call Butterfly Spread: Different Holding Periods



At low stock prices, the time value will be greatest on the call with the lowest exercise price. Since we are also long that call, we can sell it back early and recapture some of its remaining time value. This advantage also decreases, however, as we hold the position longer and time value decays.

In the middle range of stock prices, the time value will be very high on the two short calls. For short holding periods, this is a disadvantage because we have to buy back these calls, which means that we must pay for the remaining time value. This disadvantage turns to an advantage, however, as the holding period lengthens and time value begins to disappear. At expiration, no time value remains; thus, profit is maximized in this range.

The breakeven stock prices are substantially further away with shorter holding periods. This is advantageous, because it will then take a much larger stock price change to produce a loss.

As always, we cannot specifically identify an optimal holding period. Because the butterfly spread is one in which the trader expects the stock price to stay within a narrow range, profit is maximized with a long holding period. The disadvantage of a long holding period, however, is that it gives the stock price more time to move outside of the profitable range.

All of the above spreads are money spreads. We now turn to an examination of calendar spreads.

CALENDAR SPREADS

A calendar spread, also known as a time spread or horizontal spread, involves the purchase of an option with one expiration date and the sale of an otherwise identical option with a different expiration date. Because it is not

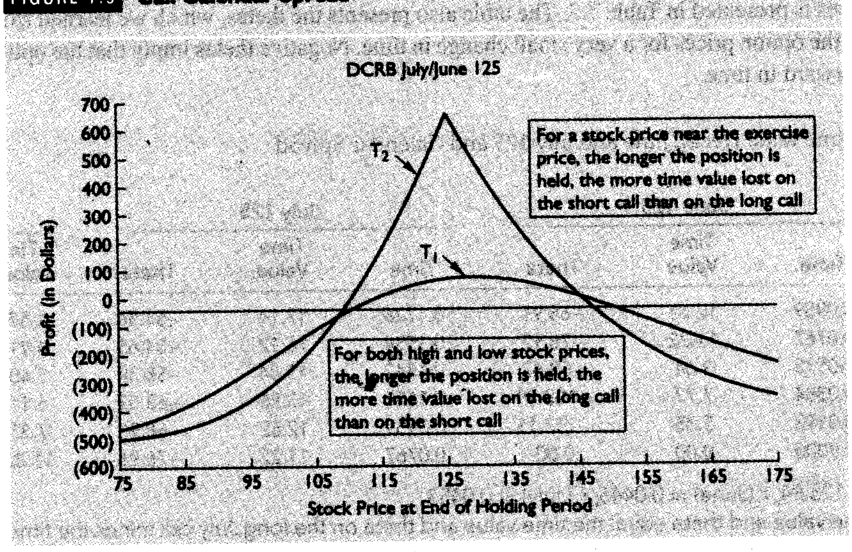
[1]The holding period was changed in this example because the time value decay does not show up as clearly for the holding periods we have previously used.

possible to hold both options until expiration, analyzing a calendar spread is more complicated than analyzing a money spread. Since one option expires before the other, the longest possible holding period would be to hold the position until the shorter-maturity option's expiration. Then, the other option would have some remaining time value that must be estimated.

Because both options have the same exercise prices, they will have the same intrinsic values; thus, the profitability of the calendar spread will be determined solely by the difference in their time values. The longer-term call will have more time value. This does not, however, necessarily mean that one should always buy the longer-term call and sell the shorter-term call. As with most option strategies, which option is purchased and which one is sold depends on the investor's outlook for the stock.

To best understand the calendar spread, we will again illustrate with the DCRB calls. This spread consists of the purchase of the July 125 call at $18.60 and the sale of the June 125 call at $13.50. This position is net long, because you pay more for the July than you receive for the June. Consider two possible holding periods. One, T_1, will involve the spread's termination on June 1; the other, T_2, will have the spread held until June 18, the date of the June call's expiration. Using the Black-Scholes-Merton model to estimate the remaining time values produced the graph in Figure 7.9.

FIGURE 7.9 **Call Calendar Spread**



Like the butterfly spread, the calendar spread is one in which the stock's volatility is the major factor in its performance. The investor obtains the greatest profit if the stock has low volatility and thus trades within a narrow range. If the stock price moves substantially, the investor will likely incur a loss.

How does the calendar spread work? Recall that we are short the June call and long the July call. When closing out the position, we buy back the June call and sell the July call. If the stock price is around the exercise price, both calls will have more time value remaining than if the stock price were at the extremes. The June call will always have less time value, however, than the July call on any given date. Thus, when we close out the position, the time value repurchased on the June call will be low relative to the remaining time value received from the sale of the July call. As we hold the position closer and closer to the June call's expiration, the remaining time value we must repurchase on that option will get lower and lower.

If the stock price is at the high or low extreme, the time values of both options will be low. If the stock price is high enough or low enough, there may be little, if any, time value on either option. Thus, when closing out the position there may be little time value to recover from the July option. Because the July call is more expensive, we will end up losing money on the overall transaction.

The breakeven stock prices can be obtained only by visual examination.[2] In this example, the shortest holding period has a tighter range between its two breakeven stock prices, about \$115 and \$145. For the longer holding period, the lower breakeven is about \$110 and the higher breakeven is still around \$145.

An investor who expected the stock price to move into the extremes could execute a reverse calendar spread. This would require purchasing the June call and selling the July call. If the stock price became extremely low or high, there would be little time value remaining to be repurchased on the July call. Because the spreader received more money from the sale of the July call than was paid for the purchase of the June call, a profit would be made. If the stock price ended up around the exercise price, however, the trader could incur a potentially large loss. This is because the July call would possibly have a large time value that would have to be repurchased.

Time Value Decay

Because a calendar spread is completely influenced by the behavior of the two calls' time value decay, it provides a good opportunity to examine how time values decay. Using the Black-Scholes-Merton model, we can compute the week-by-week time values for each call during the spread's life, holding the stock price constant at \$125.94. Keep in mind, of course, that time values will change if the stock price changes. Because time values are greatest for at-the-money options, we use the June and July 125 calls. The pattern of time values at various points during the options' lives is presented in Table 7.2. The table also presents the thetas, which we learned from Chapter 5 are the changes in the option prices for a very small change in time. Negative thetas imply that the option price will fall as we move forward in time.

Table 7.2 Time Value Decay, June and July 125 and Calendar Spread

	June 125			July 125			Spread	
Date	Time	Time Value	Theta	Time	Time Value	Theta	Time Value	Theta
May 14	0.0959	12.61	−68.91	0.1726	17.14	−51.54	4.53	17.37
May 21	0.0767	11.22	−76.92	0.1534	16.12	−54.65	4.91	22.27
May 28	0.0575	9.64	−88.63	0.1342	15.04	−58.39	5.40	30.24
June 4	0.0384	7.77	−108.19	0.1151	13.88	−63.02	6.11	45.17
June 11	0.0192	5.35	−152.11	0.0959	12.62	−68.95	7.27	83.16
June 18	0.0000	0.00	0.00	0.0767	11.22	−76.96	11.22	−76.96

$\sigma = 0.83$, $S_0 = 125.94$, $r_c(\text{June}) = 0.0446$, $r_c(\text{July}) = 0.0453$
The spread time value and theta equal the time value and theta on the long July call minus the time value and theta on the short June call. The time value of each option is obtained as the Black-Scholes-Merton value, which is the intrinsic value plus the time value, minus the intrinsic value of 0.94.

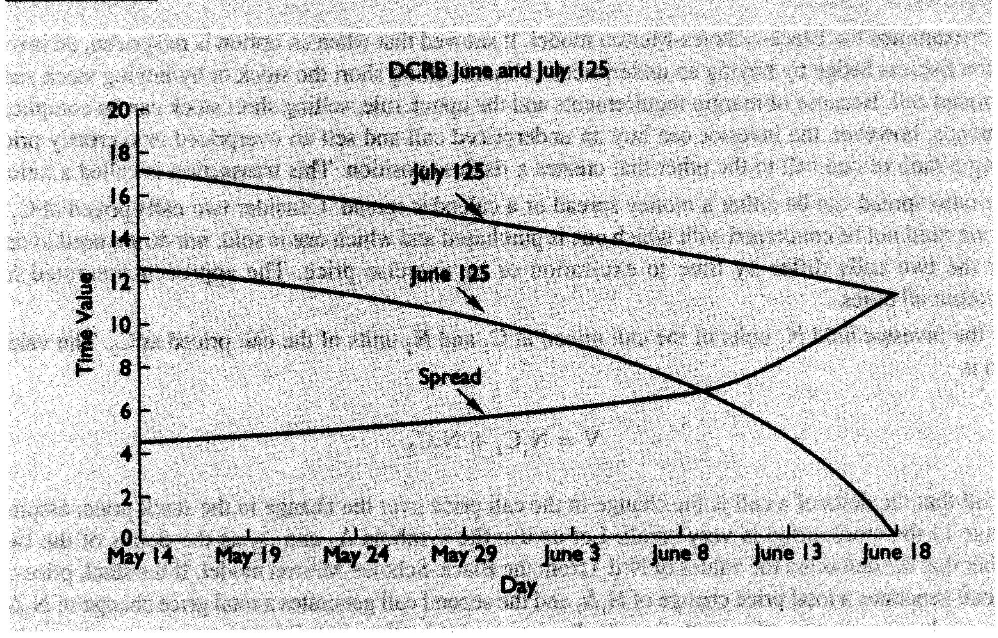
Notice what happens as expiration approaches. Because of the June call's earlier expiration, its theta is more negative, and its time value decays more rapidly than does that of the July call. Since we are long the July call and short the June call, the spread's time value—the time value of the long call minus the time value of the short call—increases, as is indicated by its positive theta. Once the June call expires, however, we are left with a long position in the July call, which leaves us with a negative theta. Time value will then begin decaying.

Figure 7.10 illustrates the pattern of time value decay. As expiration approaches, the time value of the June call rapidly decreases and the overall time value of the spread increases. At expiration of the June call, the spread's time value is composed entirely of the July call's time value.

Time value decay would appear to make it easy to profit with a time spread. One would simply buy the longer-term option and write the shorter-term option. As the time values decayed, the spread would gain value. In reality,

[2]It is possible, however, to use a computer search routine with the Black-Scholes-Merton model to find the precise breakeven.

FIGURE 7.10 Time Value Decay



however, it seldom works out like this. The pattern of time value decay illustrated here was obtained by holding the stock price constant. In the real world, the stock price will almost surely change. Thus, there is indeed risk to a calendar spread. This risk is mitigated somewhat by the fact that the investor is long one option and short the other. Nonetheless, the calendar spread, in which one buys the long-term option and writes the short-term option, is a good strategy if one expects the stock price to remain fairly stable.

The degree of risk of early exercise on a calendar spread depends on which call is bought and which is sold. Since both calls have the same exercise price, the extent to which they are in-the-money is the same. As discussed in Chapter 3, however, the time to expiration is a factor in encouraging early exercise. We saw that if everything else is equal, the shorter-term option is the one more likely to be exercised early. Thus, if we write the shorter-term option, it could be exercised early. We always, however, have the choice of exercising the longer-term option early. It will certainly be in-the-money if the shorter-term option is in-the-money. If S_t is the stock price prior to expiration and the shorter-term call is exercised, the cash flow will be $-(S_T - X)$, while the cash flow from exercising the longer-term option will be $S_T - X$. Thus, the total cash flow will be zero. This means that in the event of early exercise there will be no negative cash flow. It does not mean that there will be no overall loss on the transaction. The longer-term call is more expensive than the shorter-term call. This means that early exercise would ensure a loss by preventing us from waiting to capture the time value decay.

Calendar spreads can also be constructed with puts. By using the Black-Scholes-Merton model to determine the time value on the puts, similar results can be obtained. Like money spreads, put calendar spreads should not be overlooked. The puts could be mispriced, in which case a spread might offer the most profitable opportunity.

The butterfly spread and calendar spread are two of several transactions called volatility strategies. We shall look at the others later in this chapter. For now, however, note that all the strategies covered so far are risky. Some option traders prefer riskless strategies because if the options are mispriced, it may be possible to construct a riskless portfolio that will earn a return in excess of the risk-free rate.

RATIO SPREADS

Chapter 5 examined the Black-Scholes-Merton model. It showed that when an option is mispriced, an investor can construct a riskless hedge by buying an underpriced call and selling short the stock or by buying stock and selling an overpriced call. Because of margin requirements and the uptick rule, selling short stock can be complicated. By using spreads, however, the investor can buy an underpriced call and sell an overpriced or correctly priced call, producing a ratio of one call to the other that creates a riskless position. This transaction is called a ratio spread.

The ratio spread can be either a money spread or a calendar spread. Consider two calls priced at C_1 and C_2. Initially we need not be concerned with which one is purchased and which one is sold, nor do we need to determine whether the two calls differ by time to expiration or by exercise price. The approach presented here can accommodate all cases.

Let the investor hold N_1 units of the call priced at C_1 and N_2 units of the call priced at C_2. The value of the portfolio is

$$V = N_1C_1 + N_2C_2.$$

Recall that the delta of a call is the change in the call price over the change in the stock price, assuming that the change in the stock price is very small. Let us use the symbols Δ_1 and Δ_2 as the deltas of the two calls. Remember that the deltas are the values of $N(d_1)$ from the Black-Scholes-Merton model. If the stock price changes, the first call generates a total price change of $N_1\Delta_1$ and the second call generates a total price change of $N_2\Delta_2$. Thus, when the stock price changes, the portfolio will change in value by the sum of these two values. A hedged position is one in which the portfolio value will not change when the stock price changes. Thus, we set $N_1\Delta_1 + N_2\Delta_2$ to zero and solve for the ratio N_1/N_2.

$$\frac{N_1}{N_2} = -\frac{\Delta_2}{\Delta_1}$$

A riskless position is established if the ratio of the quantity of the first call to the quantity of the second call equals minus the inverse ratio of their deltas. The transaction would then be delta neutral.

Consider an example using the DCRB June 120 and June 125 calls. Using $S_0 = 125.94$, $r_c = 0.0446$, and $T = 0.0959$ in the Black-Scholes-Merton model gives a value of $N(d_1)$ of 0.630 for the June 120 and 0.569 for the June 125. Thus, the ratio of the number of June 120s to June 125s should be $-(0.569/0.630) = -0.903$. Hence, the investor would buy 903 of the June 120s and sell 1,000 of the June 125s.

Note that the investor could have purchased 1,000 of the June 125s and sold 903 of the June 120s. The negative sign in the formula is a reminder to be long one option and short the other. An investor should, of course, always buy underpriced or correctly priced calls and sell overpriced or correctly priced calls.

If the stock price decreases by \$1, the June 120 should decrease by 0.630 and the June 125 by 0.569. The investor is long 903 of the June 120s and therefore loses $0.630(903) \approx 569$. Likewise, the investor is short 1,000 of the June 125s and thus gains $0.569(1{,}000) \approx 569$. The gain on one call offsets the loss on the other.

The ratio spread, of course, does not remain riskless unless the ratio is continuously adjusted. Because this is somewhat impractical, no truly riskless hedge can be constructed. Moreover, the values of $N(d_1)$ are simply approximations of the change in the call price for a change in the stock price. They apply for only very small changes in the stock price. For larger changes in the stock price, the hedger would need to consider the gamma, which we discussed in Chapter 5. Nonetheless, spreads of this type are frequently done by option traders attempting to replicate riskless positions. Although the positions may not always be exactly riskless, they will come very close to being so as long as the ratio does not deviate too far from the optimum.

This completes our coverage of option spread strategies. The next group of strategies is called combinations, because they involve combined positions in puts and calls. We previously covered some combination strategies, namely conversions and reversals, which we used to illustrate put-call parity. The strategies covered in the remainder of this chapter are straddles and box spreads. We will use the same approach as before; the notation should be quite familiar by now.

STRADDLES

Straddles, like calendar and butterfly spreads, are volatility strategies because they are based on the expectation of high or low volatility rather than the direction of the stock.

A straddle is the purchase of a call and a put that have the same exercise price and expiration date. By holding both a call and a put, the trader can capitalize on stock price movements in either direction.

Consider the purchase of a straddle with the call and put having an exercise price of X and an expiration of T. Then $N_c = 1$ and $N_p = 1$ and the profit from this transaction if held to expiration is

$$\Pi = \text{Max}(0, S_T - X) - C + \text{Max}(0, X - S_T) - P.$$

Since there is only one exercise price involved, there are only two ranges of the stock price at expiration. The profits are as follows:

$$\Pi = S_T - X - C - P \quad \text{if } S_T \geq X.$$
$$\Pi = X - S_T - C - P \quad \text{if } S_T < X.$$

For the first case, in which the stock price equals or exceeds the exercise price, the call expires in-the-money.[3] It is exercised for a gain of $S_T - X$ while the put expires out-of-the-money. The profit is the gain on the call minus the premiums paid on the call and the put. For the second case, in which the stock price is less than the exercise price, the put expires in-the-money and is exercised for a gain of $X - S_T$. The profit is the gain on the put minus the premiums paid for the put and the call.

For the range of stock prices above the exercise price, the profit increases dollar for dollar with the stock price at expiration. For the range of stock prices below the exercise price, the profit decreases dollar for dollar with the stock price at expiration. When the options expire with the stock price at the exercise price, both options are at-the-money and essentially expire worthless. The profit then equals the premiums paid, which, of course, makes it a loss. These results suggest that the graph is V-shaped. Figure 7.11 illustrates the straddle for the DCRB June 125 options. The dashed lines are the strategies of buying the call and the put separately.

As noted, the straddle is designed to capitalize on high stock price volatility. To create a profit, the stock price must move substantially in either direction. It is not necessary to know which way the stock will go; it is necessary only that it make a significant move. How much must it move? Look at the two breakeven points.

For the case in which the stock price exceeds the exercise price, set the profit equal to zero:

$$S_T^* - X - C - P = 0.$$

Solving for S_T^* gives a breakeven of

$$S_T^* = X + C + P$$

The upside breakeven is simply the exercise price plus the premiums paid for the options.

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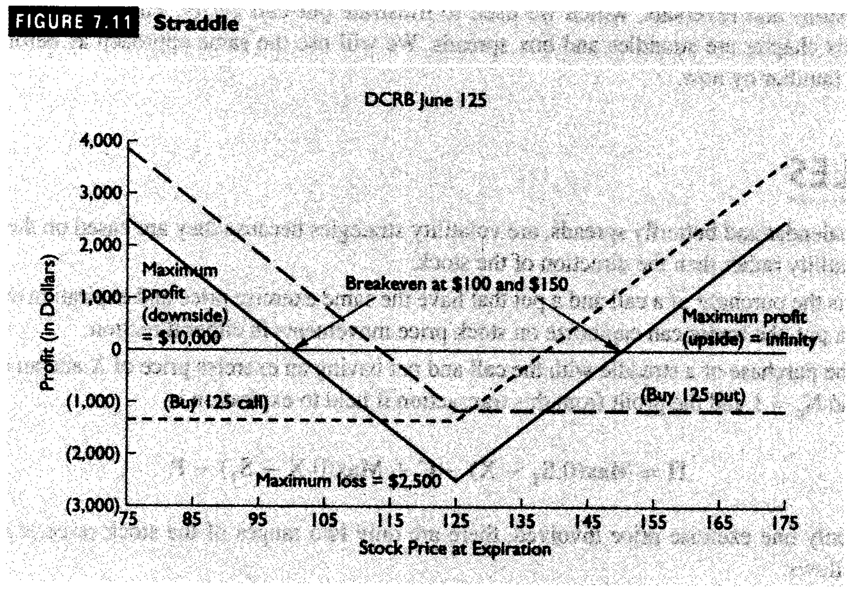
[3]The case in which $S_T = X$ is included in this range. Even though $S_T = X$ means that the call is at-the-money, it can still be exercised for a gain of $S_T - X = 0$.

FIGURE 7.11 Straddle



For the case in which the stock price is below the exercise price, set the profit equal to zero:

$$X - S_T^* - C - P = 0$$

Solving for S_T^* gives a breakeven of

$$S_T^* = X - C - P$$

The downside breakeven is the exercise price minus the premiums paid on the options.

Thus, the breakeven stock prices are simply the exercise price plus or minus the premiums paid for the call and the put. This makes sense. On the upside, the call is exercised for a gain equal to the difference between the stock price and the exercise price. For the investor to profit, the stock price must exceed the exercise price by enough that the gain from exercising the call will cover the premiums paid for the call and the put. On the downside, the put is exercised for a gain equal to the difference between the exercise price and the stock price. To generate a profit, the stock price must be sufficiently below the exercise price that the gain on the put will cover the premiums on the call and the put.

In this example, the premiums are \$13.50 for the call and \$11.50 for the put for a total of \$25. Thus, the breakeven stock prices at expiration are \$125 plus or minus \$25, or \$100 and \$150. The stock price currently is at \$125.94. To generate a profit, the stock price must increase by \$24.06 or decrease by \$25.94 in the remaining 35 days until the options expire.

The worst-case outcome for a straddle is for the stock price to end up equal to the exercise price where neither the call nor the put can be exercised for a gain.[4] The option trader will lose the premiums on the call and the put, which in this example total 100(13.50 + 11.50) = 2,500.

The profit potential on a straddle is unlimited. The stock price can rise infinitely, and the straddle will earn profits dollar for dollar with the stock price in excess of the exercise price. On the downside, the profit is limited simply because the stock price can go no lower than zero. The downside maximum profit is found by setting the stock price at expiration equal to zero for the case in which the stock price is below the exercise price. This gives a profit of X − C − P, which here is 100(125 − 13.50 − 11.50) = 10,000.

[4]Either the put, the call, or both could be exercised, but the gain on either would be zero. Transaction costs associated with exercise would suggest that neither the call nor the put would be exercised when $S_T = X$.

The potentially large profits on a straddle can be a temptation too hard to resist. One should be aware that the straddle normally requires a fairly large stock price move to be profitable. Even to a novice investor stock prices always seem highly volatile, but that volatility may be misleading. In this example, it would require about a 19 percent increase or a 21 percent decrease in the stock price in one month to make a profit, which would be a rare event. An investor considering a straddle is advised to carefully assess the probability that the stock price will move into the profitable range.

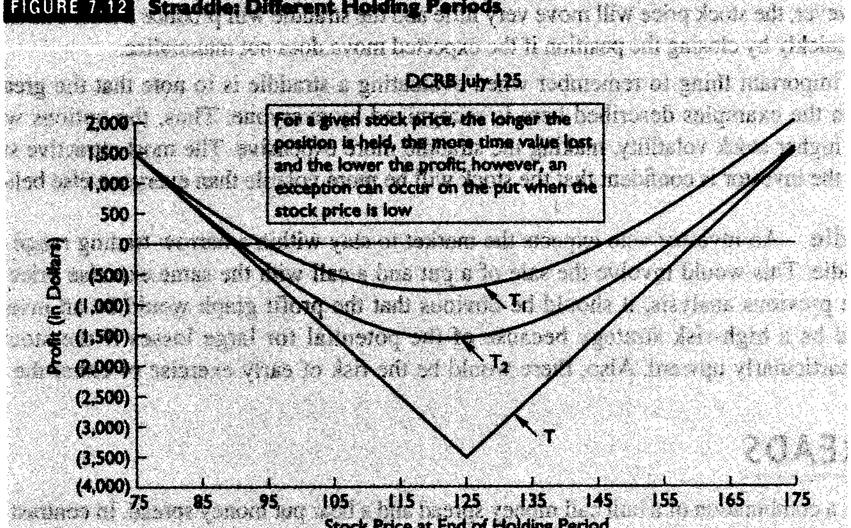
Because both the call and the put are owned, the problem of early exercise does not exist with a straddle. The early-exercise decision is up to the straddle holder. Transaction costs also need to be considered.

When the straddle is established, there is a commission on both the call and the put. At exercise there will be a commission only on either the call or the put, whichever is in-the-money. Suppose that the stock price ends up slightly higher than the exercise price. Because of the commission on the exercise of the call, it might be inadvisable to exercise the call even though it is in-the-money. A similar argument can be made for the case against exercising the put when the stock price ends up slightly less than the exercise price. This means that, as with any option strategy, the maximum loss is slightly more than the analysis indicates because of the commission. Moreover, the stock price at which such a loss occurs is actually a range around the exercise price.

Choice of Holding Period Now consider what happens upon closing the position prior to expiration. Figure 7.12 illustrates the outcomes for the DCRB July 125 straddle using the same three holding periods employed in examining the other strategies;[5] that is, the shortest holding period involves closing the position on June 4, the intermediate-length holding period on June 25, and the long holding period at expiration. The profit graphs are curves that collapse onto the straight line for the case in which the position is held to expiration. The highest curve is the shortest holding period.

We should keep in mind that this graph does not imply that the shortest holding period is the best strategy. For a given stock price, the shortest holding period indeed provides the highest profit. The uncertainty of the stock price at expiration prevents the short holding period from dominating the longer holding periods. Because a straddle is designed to permit profiting from large stock price fluctuations, the short holding period leaves less time for the stock price to make a significant move.

FIGURE 7.12 Straddle: Different Holding Periods



[5]Specifically, the holding period T_1 means that the remaining maturity is 0.1151 based on 42 days remaining, T_2 means that the remaining maturity is 0.0575 based on 21 days remaining, and $T = 0$ assumes holding all the way to expiration.

When the straddle is closed out prior to expiration, both the call and the put will contain some remaining time value. If the stock price is extremely high or low, neither option will have much time value, but either the call or the put will have a high intrinsic value. If the stock price is close to the exercise price, both options will have a fair amount of time value. When closing out the position, the investor sells the options back and recovers this time value. As the holding period is extended closer to the expiration date, there is less time value to recover and the profit declines. The profit curve gradually decreases until at expiration it becomes the curve for the case in which the straddle is held to expiration. Thus, the higher profits from shorter holding periods come from recapturing the time value of the put and the call.

Figure 7.12 shows that the shorter holding period leads to a lower upside and higher downside breakeven. This reduces the risk to the trader, because the range of stock prices in which a loss can be incurred is smaller. In this example, the shortest holding period has breakeven stock prices of about $93 and $146, and the intermediate-term holding period has breakeven stock prices of about $90 and $156.

Applications of Straddles A straddle is an appropriate strategy for situations in which one suspects that the stock price will move substantially but does not know in which direction it will go. An example of this occurs when a major bank or corporation is about to fail.

Suppose that a failing bank applies for a bailout from the government.[6] During the period in which the request is under consideration, a straddle will be a potentially profitable strategy. If the request is denied, the bank probably will fail and the stock will become worthless. If the bailout is granted, the bank may be able to turn itself around, in which case the stock price will rise substantially.

A similar scenario exists when a major corporation applies for federal loan guaranties. A straddle is also an appropriate strategy for situations in which important news is about to be released and it is expected that it will be either very favorable or very unfavorable. The weekly money supply announcements present opportunities that possibly could be exploited with index options. Corporate earnings announcements are other examples of situations in which uncertain information will be released on a specific date.

The straddle certainly is not without risk. If investors already know or expect the information, the stock price may move very little when the announcement is made. If this happens, the investor might be tempted to hold on to the straddle in the faint hope that some other, unanticipated news will be released before the options expire. In all likelihood, however, the stock price will move very little and the straddle will produce a loss. The trader might wish to cut the loss quickly by closing the position if the expected move does not materialize.

The most important thing to remember when evaluating a straddle is to note that the greater uncertainty associated with the examples described here is recognized by everyone. Thus, the options would be priced according to a higher stock volatility, making the straddle more expensive. The most attractive straddles will be those in which the investor is confident that the stock will be more volatile than everyone else believes.

Short Straddle An investor who expects the market to stay within a narrow trading range might consider writing a straddle. This would involve the sale of a put and a call with the same exercise price and expiration date. From the previous analysis, it should be obvious that the profit graph would be an inverted V. A short straddle would be a high-risk strategy because of the potential for large losses if the stock price moved substantially, particularly upward. Also, there would be the risk of early exercise of either the put or the call.

BOX SPREADS

A box spread is a combination of a bull call money spread and a bear put money spread. In contrast to the volatility strategies, the box spread is a low-risk—in fact, riskless—strategy.

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[6]Bailouts frequently take the form of loan guaranties but can also involve sale of unproductive assets, or sale of new equity or hybrid securities.

Consider a group of options with two exercise prices, X_1 and X_2, and the same expiration. A bull call spread would involve the purchase of the call with exercise price X_1 at a premium of C_1 and the sale of the call with exercise price X_2 at a premium of C_2. A bear put spread would require the purchase of the put with exercise price X_2. at a premium of P_2 and the sale of the put with exercise price X_1 at a premium of P_1. Under the rules for the effect of exercise price on put and call prices, both the call and put spread would involve an initial cash outflow, because $C_1 > C_2$ and $P_2 > P_1$. Thus, the box spread would have a net cash outflow at the initiation of the strategy.

The profit at expiration is

$$\Pi = \text{Max}(0, S_T - X_1) - C_1 - \text{Max}(0, S_T - X_2) + C_2 + \text{Max}(0, X_2 - S_T) - P_2 - \text{Max}(0, X_1 - S_T) + P_1$$

Because there are two exercise prices, we must examine three ranges of the stock price at expiration. The profits are

$$\Pi = -C_1 + C_2 + X_2 - S_T - P_2 - X_1 + S_T + P_1$$
$$= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 \qquad \text{if } S_T \leq X_1 < X_2.$$
$$\Pi = S_T - X_1 - C_1 + C_2 + X_2 - S_T - P_2 + P_1$$
$$= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 \qquad \text{if } X_1 < S_T \leq X_2.$$
$$\Pi = S_T - X_1 - C_1 + X_2 - S_T + C_2 - P_2 + P_1$$
$$= X_2 - X_1 - C_1 + C_2 - P_2 + P_1 \qquad \text{if } X_1 < X_2 < S_T.$$

Notice that the profit is the same in each case: The box spread will be worth $X_2 - X_1$ at expiration, and the profit will be $X_2 - X_1$ minus the premiums paid, $C_1 - C_2 + P_2 - P_1$. The box spread is thus a riskless strategy. Why would anyone want to execute a box spread if one can more easily earn the risk-free rate by purchasing Treasury bills? The reason is that the box spread may prove to be incorrectly priced, as a valuation analysis can reveal.

Because the box spread is a riskless transaction that pays off the difference in the exercise prices at expiration, it should be easy to determine whether it is correctly priced. The payoff can be discounted at the risk-free rate. The present value of this amount is then compared to the cost of obtaining the box spread, which is the net premiums paid. This procedure is like analyzing a capital budgeting problem. The present value of the payoff at expiration minus the net premiums is a net present value (NPV). Since the objective of any investment decision is to maximize NPV, an investor should undertake all box spreads in which the NPV is positive. On those spreads with a negative NPV, one should execute a reverse box spread.

The net present value of a box spread is

$$NPV = (X_2 - X_1)(1 + r)^{-T} - C_1 + C_2 - P_2 + P_1,$$

where r is the risk-free rate and T is the time to expiration.[7] If NPV is positive, the present value of the payoff at expiration will exceed the net premiums paid. If NPV is negative, the total amount of the premiums paid will exceed the present value of the payoff at expiration. The process is illustrated in Figure 7.13.

An alternative way to view the box spread is as the difference between two put-call parities. For example, for the options with exercise price X_1, put-call parity is

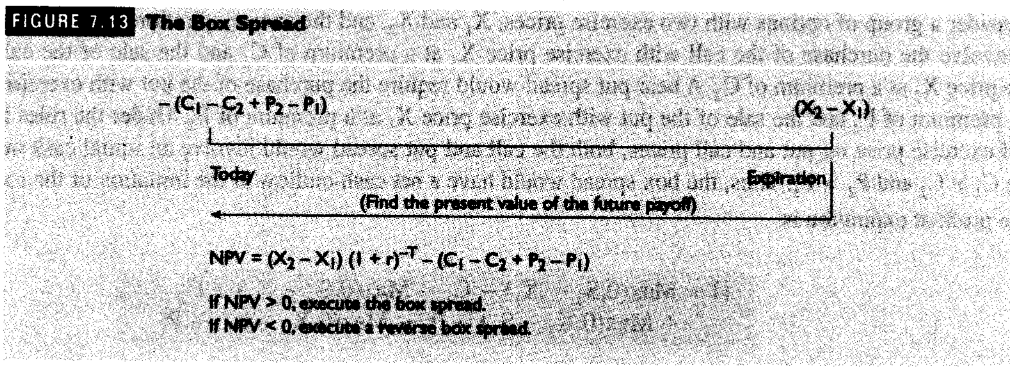
$$P_1 = C_1 - S_0 + X_1(1 + r)^{-T},$$

and for the options with exercise price X_2, put-call parity is

$$P_2 = C_2 - S_0 + X_2(1 + r)^{-T}.$$

[7]Alternately, one could compute the present value of $X_2 - X_1$ as $(X_2 - X_1)e^{-r_c T}$ and obtain the same result.

FIGURE 7.13 The Box Spread



Rearranging both equations to isolate the stock price gives

$$S_0 = C_1 - P_1 + X_1(1 + r)^{-T}.$$
$$S_0 = C_2 - P_2 + X_2(1 + r)^{-T}.$$

Since the left-hand sides are equal, the right-hand sides must also be equal; therefore,

$$C_1 - P_1 + X_1(1 + r)^{-T} = C_2 - P_2 + X_2(1 + r)^{-T}.$$

Rearranging this equation gives

$$0 = (X_2 - X_1)(1 + r)^{-T} - C_1 + C_2 - P_2 + P_1.$$

This is our put-call parity equation when the NPV is zero, which results if all puts and calls are correctly priced relative to one another.

Let us examine the DCRB June box spread using the 125 and 130 options. Consider the following transaction: Buy the 125 call at $13.50, buy the 130 put at $14.25, write the 130 call at $11.35, and write the 125 put at $11.50. The premiums paid for the 125 call and 130 put minus the premiums received for the 130 call and 125 put net out to $4.90. Thus, it will cost $490 to buy the box spread.

The payoff at expiration is $X_2 - X_1$. The net present value is

$$NPV = 100[(130 - 125)(1.0456)^{-0.0959} - 4.90] = \$7.85,$$

where 0.0456 is the discrete risk-free rate for June as determined in Chapter 3, and 0.0959 is the time to expiration from May 14 to June 18. Thus, the spread is underpriced and should be purchased. Had the NPV been negative, the box spread would have been overpriced and should be sold. In that case, an investor would buy the 130 call and 125 put and sell the 125 call and 130 put. This would generate a positive cash flow up front that exceeded the present value of the cash outflow of $X_2 - X_1$ at expiration.

If the investor is holding a long box spread, the risk of early exercise is unimportant. Suppose that the short call is exercised. Because the short call is in-the-money, the long call will be even deeper in-the-money. The investor can then exercise the long call. If the short put is exercised, the investor can in turn exercise the long put, which will be even deeper in-the-money than the short put. The net effect is a cash inflow of $X_2 - X_1$ the maximum payoff at expiration. For the short box spread, however, early exercise will result in a cash outflow of $X_2 - X_1$. Thus, the early exercise problem is an important consideration for short box spreads.

Transaction costs on a box spread will be high because four options are involved. At least two of the four options, however, will expire out-of-the-money. Nonetheless, the high transaction costs will make the box spread costly to execute for all but those who own seats on the exchange.

QUESTIONS AND PROBLEMS

1. Suppose that you are following the stock of a firm that has been experiencing severe problems. Failure is imminent unless the firm is granted government-guaranteed loans. If the firm fails, its stock will, of course, fall substantially. If the loans are granted, it is expected that the stock will rise substantially. Identify two strategies that would be appropriate for this situation. Justify your answers.
2. Explain how a short call added to a protective put forms a collar and how it changes the payoff and up-front cost.
3. Derive the profit equations for a put bull spread. Determine the maximum and minimum profits and the breakeven stock price at expiration.
4. Explain the process by which the profit of a short straddle closed out prior to expiration is influenced by the time values of the put and call.
5. The chapter showed how analyzing a box spread is like a capital budgeting problem using the net present value approach. Consider the internal rate of return method of examining capital budgeting problems and analyze the box spread in that context.

The following option prices were observed for calls and puts on a stock on July 6 of a particular year. Use this information for problems 6 through 20. The stock was priced at 165.13. The expirations are July 17, August 21, and October 16. The continuously compounded risk-free rates associated with the three expirations are 0.0503, 0.0535, and 0.0571, respectively. The standard deviation is 0.21.

	Calls			Puts		
Strike	**Jul**	**Aug**	**Oct**	**Jul**	**Aug**	**Oct**
160	6.00	8.10	11.10	0.75	2.75	4.50
165	2.70	5.25	8.10	2.40	4.75	6.75
170	0.80	3.25	6.00	5.75	7.50	9.00

For problems 6 through 10 and 13 through 16, determine the profits for the holding period indicated for possible stock prices of 150, 155, 160, 165, 170, 175, and 180 at the end of the holding period. Answer any other questions as indicated. Note: Your Excel spreadsheet Stratlyz7e.xls will be useful here for obtaining graphs as requested but it does not allow you to calculate the profits for several user-specified asset prices. It permits you to specify one asset price and a maximum and minimum. Use Stratlyz7e.xls to produce the graph for the range of prices from 150 to 180 but determine the profits for the prices of 150, 155, . . . , 180 by hand for positions held to expiration. For positions closed prior to expiration, use the spreadsheet BSMbin7e.xls or the Windows program BSMbwin7e.exe to determine the option price when the position is closed; then calculate the profit by hand.

6. Construct a bear money spread using the October 165 and 170 calls. Hold the position until the options expire. Determine the profits and graph the results. Identify the breakeven stock price at expiration and the maximum and minimum profits. Discuss any special considerations associated with this strategy.
7. Repeat problem 6, but close the position on September 20. Use the spreadsheet to find the profits for the possible stock prices on September 20. Generate a graph and use it to identify the approximate breakeven stock price.
8. Construct a collar using the October 160 put. First use the Black-Scholes-Merton model to identify a call that will make the collar have zero up-front cost. Then close the position on September 20. Use the spreadsheet to find the profits for the possible stock prices on September 20. Generate a graph and use it to identify the approximate breakeven stock price. Determine the maximum and minimum profits.

9. Suppose that you are expecting the stock price to move substantially over the next three months. You are considering a butterfly spread. Construct an appropriate butterfly spread using the October 160, 165, and 170 calls. Hold the position until expiration. Determine the profits and graph the results. Identify the two breakeven stock prices and the maximum and minimum profits.
10. Construct a calendar spread using the August and October 170 calls that will profit from high volatility. Close the position on August 1. Use the spreadsheet to find the profits for the possible stock prices on August 1. Generate a graph and use it to estimate the maximum and minimum profits and the breakeven stock prices.
11. Using the Black-Scholes-Merton model, compute and graph the time value decay of the October 165 call on the following dates: July 15, July 31, August 15, August 31, September 15, September 30, and October 16. Assume that the stock price remains constant. Use the spreadsheet to find the time value in all of the cases.
12. Consider a riskless spread with a long position in the August 160 call and a short position in the October 160 call. Determine the appropriate hedge ratio. Then show how a $1 stock price increase would have a neutral effect on the spread value. Discuss any limitations of this procedure.
13. Construct a long straddle using the October 165 options. Hold until the options expire. Determine the profits and graph the results. Identify the breakeven stock prices at expiration and the minimum profit.
14. Repeat problem 13, but close the positions on September 20. Use the spreadsheet to find the profits for the possible stock prices on September 20. Generate a graph and use it to identify the approximate breakeven stock prices.
15. A slight variation of a straddle is a strap, which uses two calls and one put. Construct a long strap using the October 165 options. Hold the position until expiration. Determine the profits and graph the results. Identify the breakeven stock prices at expiration and the minimum profit. Compare the results with the October 165 straddle.
16. A strip is a variation of a straddle involving two puts and one call. Construct a short strip using the August 170 options. Hold the position until the options expire. Determine the profits and graph the results. Identify the breakeven stock prices at expiration and the minimum profit.
17. Analyze the August 160/170 box spread. Determine whether a profit opportunity exists and, if so, how one should exploit it.
18. Complete the following table with the correct formula related to various spread strategies.

Item	Bull Spread With Calls	Bear Spread With Puts	Butterfly Spread With Calls
Value at expiration			
Profit			
Maximum profit			
Maximum loss			
Breakeven			and

Item	Collar Strategies With Calls and Puts	Straddle With Calls and Puts
Value at expiration		
Profit		
Maximum profit		
Maximum loss		
Breakeven		and

19. Complete the following table with the correct formula related to various spread strategies.
20. (Concept Problem) Many option traders use a combination of a money spread and a calendar spread called a *diagonal spread.* This transaction involves the purchase of a call with a lower exercise price and longer time to expiration and the sale of a call with a higher exercise price and shorter time to expiration. Evaluate the diagonal spread that involves the purchase of the October 165 call and the sale of the August 170 call. Determine the profits for the same stock prices you previously examined under the assumption that the position is closed on August 1. Use the spreadsheet to find the profits for the possible stock prices on August 1. Generate a graph and use it to estimate the breakeven stock price at the end of the holding period.
21. (Concept Problem) Another variation of the straddle is called a *strangle*. A strangle is the purchase of a call with a higher exercise price and a put with a lower exercise price. Evaluate the strangle strategy by examining the purchase of the August 165 put and 170 call. As in the problems above, determine the profits for stock prices of 150, 155, 160, 165, 170, 175, and 180. Hold the position until expiration and graph the results. Find the breakeven stock prices at expiration. Explain why one would want to use a strangle.
22. Explain why option traders often use spreads instead of simple long or short options and combined positions of options and stock.
23. Suppose that an option trader has a call bull spread. The stock price has risen substantially, and the trader is considering closing the position early. What factors should the trader consider with regard to closing the transaction before the options expire?
24. Explain why a straddle is not necessarily a good strategy when the underlying event is well known to everyone.

PART 2

Forwards, Futures, and Swaps

8

THE STRUCTURE OF FORWARD AND FUTURES MARKETS

Part One explored the world of options markets. The next five chapters look at forward, futures, and swap markets. In this chapter, we focus on forward and futures markets. A forward contract is an agreement between two parties, a buyer and a seller, that calls for the delivery of an asset at a future point in time with a price agreed upon today. A futures contract is a forward contract that has standardized terms, is traded on an organized exchange, and follows a daily settlement procedure in which the losses of one party to the contract are paid to the other party.

Forward and futures contracts have many of the characteristics of option contracts. Both provide for the sale and delivery of an asset on a later date at a price agreed upon today. An option—more specifically, a call option—gives the holder the right to forgo the future purchase of the asset. This is done, as we have seen, if the asset's price is below the exercise price. A forward or futures contract does not offer the right to forgo purchase of the asset. Like an exchange-listed option, however, a futures contract can be sold in the market prior to expiration. Like an over-the-counter option, a forward contract can be offset by creating a new forward contract.

Forward contracts, sometimes called forward commitments, are very common in everyday life. For example, an apartment lease is a series of forward contracts. The current month's use of the apartment is a spot transaction, but the two parties also have agreed to usage of the apartment for future months at a rate agreed upon today. Near and dear to the hearts of most college students (and many others) is your basic pizza delivery order, which is also a forward contract. The customer and the restaurant agree that the customer will buy the pizza at a specific price at a future point in time ("30 minutes or less"). Upon delivery, the customer must accept and pay for the pizza, even though in the interim the customer might have noticed an ad or a coupon that would make an identical pizza cost less.

Any type of contractual arrangement calling for the delivery of a good or service at a future date at a price agreed upon today is a forward contract. Neither party can legally get out of the commitment. Nonetheless, either party can enter into a new, offsetting forward contract with someone else. For example, suppose after ordering the pizza, you decide that you would prefer Chinese food. You know, however, that your neighbor next door is practically addicted to pizza so you offer to sell the pizza to your neighbor when it is delivered. You might even be able to negotiate a better price, depending on how hungry your neighbor is. In that case, you contracted to deliver the pizza to your neighbor and you agreed upon a price. You are long a forward contract with the restaurant and short a forward contract with your neighbor. In the case of the apartment lease, subleasing your apartment is a way of offsetting your forward contract with the landlord.

Of course, all of these transactions are subject to a degree of uncertainty about whether a party will perform as promised. For example, suppose the pizza delivery person shows up at your door and finds no one there because you changed your mind and went out for Chinese food. Alternatively, perhaps you accepted delivery of your pizza, but your neighbor refused to accept the pizza because he changed his mind. This potential for default is quite similar to that of the over-the-counter options market. There the buyer faced the potential default of the writer. In a forward contract, however, each party is subject to the default of the other. The pizza restaurant faces the potential that you will default. Do you face the potential that the restaurant will default by not delivering your pizza? The risk is slight but still there. Here the pizza delivery order is an example of a forward contract between an extremely creditworthy customer, the restaurant, and one with lower credit quality, yourself.

These examples are but small-scale, familiar cases of forward contracting. If we substitute agricultural products, precious metals, securities, or currencies for pizza, apartments, magazines, and airline tickets, we have the real world of big-time forward contracting. To mitigate the risk of default, many forward markets evolved into futures markets. Futures markets permit organized trading in standardized versions of these forward contracts. To reduce the risk of default, futures contracts require a daily settling of gains and losses. As we move through the chapter, we shall look at these characteristics in more detail. But first, let us see how these markets evolved.

DEVELOPMENT OF FORWARD AND FUTURES MARKETS

As noted above, forward contracts are common in everyday life. Quite naturally such contracts go back to the beginnings of commerce. For example, in medieval trade fairs merchants often contracted for deferred delivery of goods at a price agreed to in advance. Over the next few hundred years, organized spot markets for commodities began to develop in major European cities. Meanwhile, a similar market for rice developed in Japan. The characteristics of these markets were not too unlike those of today's futures markets. Modern futures markets, however, generally trace back to the formation of the Chicago Board of Trade in 1848.

Chicago Futures Markets

In the 1840s, Chicago was rapidly becoming the transportation and distribution center of the Midwest. Farmers shipped their grain from the farm belt to Chicago for sale and subsequent distribution eastward along rail lines and the Great Lakes. Due to the seasonal nature of grain production, however, large quantities of grain were shipped to Chicago in the late summer and fall. The city's storage facilities were inadequate for accommodating this temporary increase in supply. Prices fell drastically at harvest time as supplies increased and then rose steadily as supplies were consumed.

In 1848, a group of businessmen took the first step toward alleviating this problem by forming the Chicago Board of Trade (CBOT). The CBOT initially was organized for the purpose of standardizing the quantities and qualities of the grains. A few years later, the first forward contract was developed. Called a *to-arrive* contract, it provided that a farmer could agree to deliver the grain at a future date at a price determined in advance. This meant that the farmer would not ship the grain to Chicago at harvest time but could fix the price and the date at which the grain subsequently would be sold.

These to-arrive contracts proved to be a curious instrument. Speculators soon found that rather than buy and sell the grain itself they could buy and sell the contracts. In that way, they could speculate on the price of grain to be delivered at a future date and not have to worry about taking delivery of and storing the grain. Soon thereafter, the exchange established a set of rules and regulations for governing these transactions. In the 1920s, the clearinghouse was established to provide a means of offsetting or unwinding positions and guaranteeing to each party that the other would perform. By that time, most of the essential ingredients of futures contracts were in place.

In 1874, the Chicago Produce Exchange was formed and later became the Chicago Butter and Egg Board. In 1898, it was reorganized as the Chicago Mercantile Exchange, which is now the second largest futures exchange in the United States. Over the years many new exchanges were formed. The 1990s saw an explosion in the development of futures exchanges around the world. As we shall see later, the world's largest futures exchange is no longer in the United States.

Development of Financial Futures

For the first 120 years, futures exchanges offered trading in contracts on commodities such as agricultural goods and metals. Then, in 1971, the major Western economies began to allow their currency exchange rates to fluctuate. This opened the way for the formation in 1972 of the International Monetary Market (IMM), a subsidiary of the Chicago Mercantile Exchange that specializes in the trading of futures contracts on foreign currencies. These were the first futures contracts that could be called financial futures. The first interest rate futures contract appeared in 1975, when the Chicago Board of Trade originated its GNMA futures, a contract on Government National Mortgage Association pass-through certificates, whose yields reflect mortgage interest rates.

In 1976, the International Monetary Market introduced the first futures contract on a government security and a short-term financial instrument—90-day U.S. Treasury bills. This contract was actively traded for many years, but its popularity has declined somewhat, at least partly due to the remarkable success of a competing contract, the Eurodollar futures, which was launched in 1981.

In 1977, the Chicago Board of Trade started what became one of the most successful contracts of all time—U.S. Treasury bond futures. In just a few years, this instrument became the most actively traded contract, surpassing many grain futures that had traded for more than 100 years. In the 1990s, however, the Eurodollar contract surpassed the bond contract as the most actively traded futures contract in the United States.

The 1980s brought the highly successful stock index futures contract. This instrument, sometimes referred to as "pin-stripe pork bellies," has helped bridge the long-standing gap between New York's stock traders and Chicago's futures traders. Interestingly, however, the first stock index futures contract appeared not in New York or Chicago but in Kansas City. The Kansas City Board of Trade completed the formal registration process ahead of its New York and Chicago counterparts, and on February 16, 1982, launched the Value Line Index futures. The Index and Option Market, a division of the Chicago Mercantile Exchange, followed on April 21 with its S&P 500 futures contract. The New York Futures Exchange entered the game on May 6 with its New York Stock Exchange Index futures. Today, however, the Value Line and NYSE Index futures have little trading volume.

Interestingly, in the United States, the Dow Jones Industrial Average, the most well known measure of stock market activity, has had a futures contract on it only since 2001. Shortly after the first stock index futures contracts appeared in 1982, the Chicago Board of Trade attempted to start a contract on the Average, but was rebuffed in a lengthy legal process started by the Dow Jones Company. Dow Jones (DJ) stated that it did not want its name associated with speculative trading and believed that the CBOT and traders were profiting from its name. Standard and Poor's, on the other hand, has eagerly licensed its name to futures exchanges for contracts and has even developed new indices with an eye toward having futures contracts traded on them. Rebuffed by DJ, the CBOT then created a similar contract, called the Major Market Index, which traded successfully for a number of years but eventually died. It was not until the late 1990s, however, that Dow Jones decided that it should enter the index futures business and licensed its famous Average to the Chicago Board of Trade for futures trading and the American Stock Exchange for options trading. Dow Jones also began licensing other averages and creating new market averages, including a number for foreign markets, with the intention of encouraging futures trading on them. Today futures contracts on Dow Jones averages based on stocks, commodities, and electricity are found on various futures exchanges.

Stock index futures contracts have been extremely successful worldwide. Almost every major country has its own futures exchange with a successful stock index contract. Countries in which stock index futures are extremely popular include the United Kingdom, France, Japan, Germany, Korea, Spain, and China (Hong Kong).

In Part One, we studied both options on individual stocks and stock index options. Individual stock futures, often called single stock futures, are a fairly new instrument. These instruments trade actively in a number of countries outside the U.S., but until 2001, U.S. futures markets could not offer single stock futures because of regulatory restrictions. In Chapter 2, we noted that the Securities and Exchange Commission (SEC), which regulates stocks, and the Commodity Futures Trading Commission (CFTC), which regulates futures, had reached an agreement in 1982 on how to divide up their regulatory responsibilities. The CFTC regulates the stock index futures market, and the SEC regulates the securities and stock and stock index options markets. Single stock futures were prohibited because the regulatory authority was not clear. As futures contracts, single stock futures would ordinarily be regulated by the CFTC, but the SEC claimed that these instruments would be close substitutes for securities and single stock options. Finally, in 2001, a new law permitted these instruments and defined a joint regulatory structure. Single stock futures were then introduced in the United States in November 2002.

Over the years there has been a tremendous degree of competition between the futures exchanges to introduce new contracts that would generate significant trading volume. Barely a month passes without at least one new futures contract being introduced. A few of these contracts, such as municipal bond futures, were moderately successful. Some, such as oil futures, were highly successful. Most of the contracts, like inflation futures, commercial paper futures, and corporate bond index futures, failed to attract much trading volume. Even the original GNMA futures contract died and attempts to modify and revive it failed. You should note, however, that such failures are not a sign of weakness but rather of a healthy and highly competitive business in which only those contracts that truly meet a need will survive.

Development of Options on Futures Markets

The futures markets not only offer futures contracts. They also offer option contracts. In 1982 the first option contracts in which the underlying is a futures contract were created in the United States. This was a five-year experimental program that was deemed a tremendous success, and the instrument was permanently authorized in the United States in 1987. Options on futures have since been offered on many markets around the world and most have been relatively successful.

Options on futures are also sometimes called commodity options and more commonly, futures options. An option on futures permits the holder to buy for a call, or to sell for a put, a specific underlying futures contract at a fixed price up to a specific expiration day. The buyer of the option pays a premium and receives the call or put, which permits exercise into the underlying futures contract. Note that in this case, the option is a derivative on a derivative. That is, the underlying itself is a derivative. Thus, there are two expirations, the option's expiration and the futures' expiration. For some contracts, the option and futures expire simultaneously, effectively making the option on the futures equivalent to an option on the underlying spot asset. For all other contracts, the option expires before but relatively close to the expiration of the futures.

As noted, options on futures contracts have been very successful. This success was primarily a result of the success of options on futures in the United States, where regulations had prohibited the trading of options in the same markets in which the underlying asset would be traded. That is, options on stocks trade in one market, but the underlying stocks trade elsewhere. When options on futures were introduced, the option would trade on the same exchange on which the underlying futures traded. The parallel trading of the option and the underlying created a strong demand for arbitrage trading between these two instruments and led to highly active and efficient markets.

Parallel Development of Over-the-Counter Markets

The most active early forward market was the market for foreign exchange, called the interbank market. This market grew tremendously in response to the floating of currencies in the early 1970s, as mentioned above. It consists of hundreds of banks worldwide who make forward and spot commitments with each other, representing either themselves or their clients. The market is quite large, though the exact size is difficult to estimate, since the transactions are essentially private and unregulated. The transaction sizes are quite large as well and it would be unusual for individual investors to be able to participate in this market.

Forward markets for various financial instruments and commodities have also developed in recent years. The decade of the 1980s saw a tremendous jump in the level of understanding and appreciation for derivative instruments. While futures and options markets were growing, forward markets began to grow as well. The primary stimulant for forward market growth was the development of swaps. A swap is an agreement between two parties to exchange a series of payments. There are quite a few variations on this basic theme, but in general swaps are similar to forward contracts. We shall cover swaps in detail in Chapter 12. For now, however, let us note that the growing acceptance of swaps stimulated the development of other over-the-counter transactions, such as options as noted in Chapter 2, and a variety of other forward contracts. For example, one can enter into a forward contract, called a forward rate agreement or FRA, that is simply an arrangement for one party to pay a certain fixed amount of cash while the other party pays an amount of cash determined by the interest rate at a predetermined future date. This contract can be used to hedge or speculate on interest rates. It does not actually require delivery of a security or commodity as it is settled by simply exchanging cash. It is also possible to arrange for forward delivery of almost any security or commodity at a price agreed upon today. The forward market is a large and healthy one that competes with and, yet, complements the futures market.

OVER-THE-COUNTER FORWARD MARKET

In Chapter 2 we discussed how over-the-counter options differ from exchange-traded options. Now we shall do the same for forward and futures contracts. The forward market is large and worldwide. Its participants are banks, corporations, and governments. The two parties to a forward contract must agree to do business with each other, which means that each party accepts credit risk from the other. That is, unlike in options markets where the writer does not assume any credit risk from the buyer, in forward markets each party accepts the credit risk of the other. In spite of the credit risk, however, forward contracts offer many advantages.

The primary advantage is that the terms and conditions are tailored to the specific needs of the two parties. Suppose that a firm would like to secure the future purchase price of 400,000 bushels of sorghum, a grain similar in use to corn. As we shall see later, the futures markets permit trading only in contracts on specific commodities and with certain expiration dates. There is no sorghum futures contract that would permit the company to lock in the future purchase price of the sorghum. If the firm could substitute corn, however, there is a corn futures contract on the Chicago Board of Trade, though its expiration might not match the horizon date of the firm. Moreover, it might permit the seller of the futures to deliver any of several grades of corn at any of several locations. The firm would perhaps prefer to arrange a specific contract with the terms tailored to meet its needs. Similar arguments can be made with respect to financial contracts. A portfolio manager might wish to lock in the market value of a specific portfolio on a certain date. If the futures market does not have such a contract, the manager might look to the forward market.

As noted about the over-the-counter options market, the forward market also has the advantage of being a private market in which the general public does not know that the transaction was done. This prevents other traders from interpreting the size of various trades as perhaps false signals of information.

The over-the-counter market is also an unregulated market. Although there is now much debate about whether this market should be regulated, and we shall bring this topic up again later in this chapter, most governments view these contracts as private arrangements. This gives participants considerably more flexibility, saves money, and allows the market to quickly respond to changing needs and circumstances by developing new variations of old contracts.

Of course, all of this comes at the expense of assuming credit risk and the requirement that the transactions be of a rather large size, several million dollars or more. It is not clear which is more costly, forwards or futures, but since the markets coexist, they must be serving their clientele in an efficient manner.

How large is the market? Since the transactions are private, it is difficult to tell. A survey taken by the Bank for International Settlements of Basel, Switzerland, estimates that at year-end 2005 the size of the over-the-counter forward market is about $34 trillion of face value with an estimated market value of $541 billion.

ORGANIZED FUTURES TRADING

Futures trading is organized around the concept of a futures exchange. The exchange is probably the most important component of a futures market and distinguishes it from forward markets.

A futures exchange is a corporate entity comprised of members. Although some exchanges allow corporate memberships, most members are individuals. The members elect a board of directors, which in turn selects individuals to manage the exchange. The exchange has a corporate hierarchy consisting of officers, employees, and committees. The exchange establishes rules for its members and may impose sanctions on violators.

Although most futures exchanges are nonprofit corporations owned by their member traders, some exchanges are profit-making corporations with publicly traded stock. For example, the Chicago Mercantile Exchange and the Chicago Board of Trade common stocks trade on the New York Stock Exchange under the ticker symbols CME and BOT. It is interesting to also note that the Chicago Board Options Exchange, which we covered extensively in Chapter 2 has launched a futures exchange within itself. This exchange, called CBOE Futures, trades futures contracts on volatility. As we saw in previous chapters, volatility is a critical factor in options. These CBOE futures products permit futures trading on measures of volatility, allowing option traders to hedge the risk of changing volatility. To date, however, these contracts have not generated much trading volume.

Contract Development

One of the exchange's important ongoing activities is identifying new and useful futures contracts. Most exchanges maintain research staffs that continuously examine the feasibility of new contracts. In the United States, when the exchange determines that a contract is likely to be successful, it writes a proposal specifying the terms and conditions and applies to the Commodity Futures Trading Commission (CFTC), the regulatory authority, for permission to initiate trading.[1] Similar procedures are followed in other countries.

It is becoming increasingly difficult to determine the characteristics of an asset that make it a likely candidate for a successful futures contract. At one time, it was thought that the asset had to be storable, but there are now futures contracts on nonstorable assets such as electricity and even such factors as the weather, which is not a specific asset at all. What does seem to be a common thread is the existence of an identifiable, volatile spot price and a group of potential users who face a risk of loss if prices move in a certain direction. It is not necessary that the spot price be the price of an asset that one can actually buy and hold.

Thus, it is conceivable that virtually anything can have a futures contract traded on it. Whether the contract will be actively traded will depend on whether it fills the needs of hedgers and whether speculators are interested enough to take risks in it.

Contract Terms and Conditions

The contract's terms and conditions are determined by the exchange subject to regulatory approval. The specifications for each contract are the size, quotation unit, minimum price fluctuation, grade, and trading hours. In addition, the contract specifies delivery terms and daily price limits as well as delivery procedures, which are discussed in separate sections. Complete contract specifications can be found on the Web sites of the exchanges.

Contract size means that one contract covers a specific number of units of the underlying asset. This might be a designated number of bushels of a grain or dollars of face value of a financial instrument. Contract size is an important decision. If too small, speculators will find it more costly to trade because there is a cost for trading each contract. The contracts are not divisible; thus, if they are too large, hedgers may be unable to get a matching number of contracts. For example, if the Chicago Board of Trade had established $1 million as the Treasury bond contract, a hedger with $500,000 of bonds to hedge probably would be unable to use it.[2]

The quotation unit is simply the unit in which the price is specified. For example, corn is quoted in fourths of a cent and Treasury bonds in percentage points and thirty-seconds of a point of par value. In most cases, the spot market quotation unit is used.

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[1]The CFTC's responsibility is discussed later in the chapter.

[2]We say *probably* because if the bonds being hedged were twice as volatile as the futures contract, one contract would be the correct number. Chapter 11 explains this point.

Closely related to the quotation unit is the minimum price fluctuation. This is usually the smallest unit of quotation. For example, Treasury bonds are quoted in a minimum unit of thirty-seconds. Thus, the minimum price change on a Treasury bond futures contract is 1/32 of 1 percent of the contract price, or 0.0003125. Since the contract has a face value (contract size) of \$100,000, the minimum price change is 0.0003125 X (\$100,000) = \$31.25.

The exchange also establishes the contract grade. In the case of agricultural commodities there may be numerous grades, each of which would command a quality price differential in the spot market. The contract must specify the grades that are acceptable for delivery. Financial futures contracts must indicate exactly which financial instrument or instruments are eligible for delivery. If multiple instruments are deliverable, the seller of the contract holds a potentially valuable option, which we shall discuss in Chapter 10.

The exchange also specifies the hours during which the contract trades. Most agricultural futures trade for four to five hours during the day. Most financial futures trade for about six hours. In most U.S. markets, this trading occurs on the floor of the exchange, as described in a later section, but many exchanges have electronic trading systems, whereby trading occurs at terminals that can be placed in offices and even in homes. Electronic trading typically occurs during hours that the exchange is not open for floor trading, which in some cases means throughout the night. This practice is part of the exchange's attempt to keep pit trading alive in this age in which so much economic activity is conducted electronically. There are some contracts, however, that are designated solely for electronic trading and some others that trade simultaneously with floor trading. Some exchanges, such as the highly successful EUREX (a combined German-Swiss futures exchange) and EURONEXT (the exchange of France, the Netherlands, Belgium, Portugal, and the U.K.), have exclusively electronic trading, and trading hours are usually longer than a standard business day.

Delivery Terms

The contract must also indicate a specific delivery date or dates, the delivery procedure, and a set of expiration months. In the case of harvestable commodities, the exchange usually establishes expiration months to correspond with harvest months. In nonharvestable commodities, such as financial futures, the exchange usually has followed the pattern of allowing expirations in March, June, September, and December. There are some exceptions, however.

The exchange also decides how far into the future the expiration dates will be set. For some contracts, the expirations extend only a year or two, while the Eurodollar contract extends about ten years.

Once the expiration month has been set, the exchange determines a final trading day. This may be any day in the month, but the most common ones are the third Friday of the month and the business day prior to the last business day of the month. The first delivery day also must be set. Most contracts allow delivery on any day of the month following a particular day. Usually the first eligible delivery day is the first business day of the month, but for certain contracts other days may be specified. In the case of stock index futures and other cash-settled contracts, the settlement occurs on the last trading day or on the day after the last trading day.

For non-cash-settled contracts, the delivery procedure must be specified. The deliverable spot commodity must be sent to any of several eligible locations. Financial adjustments to the price received upon delivery are required when an acceptable but lower-grade commodity is delivered. We shall say more about the delivery procedure later.

Daily Price Limits and Trading Halts

During the course of a trading day prices fluctuate continuously, but many contracts have limits on the maximum daily price change. If a contract price hits the upper limit, the market is said to be limit up. If the price moves to the lower limit, the market is said to be limit down. Any such move, up or down, is called a limit move. Normally no transactions above or below the limit price are allowed. Some contracts have limits only during the opening minutes; others have limits that can be expanded according to prescribed rules if prices remain at the limits for extended periods.

In conjunction with price limits, some futures contracts—notably stock index futures—contain built-in trading halts sometimes called circuit breakers. When prices move rapidly, trading can be stopped for predetermined periods. These halts can be accompanied by similar halts in the spot market. Such cessations of trading were installed after the stock market crash of 1987 in response to concern that extremely volatile markets might need a cooling-off period. Although it is not clear that trading halts are necessarily effective, it seems likely that they will continue to be used in futures markets.

Other Exchange Responsibilities

The exchange also specifies that members meet minimum financial responsibility requirements. In some contracts it may establish position limits, which, like those in options markets, restrict the number of contracts that an individual trader can hold. The exchange establishes rules governing activities on the trading floor and maintains a department responsible for monitoring trading to determine whether anyone is attempting to manipulate the market. In some extreme cases, the exchange may elect to suspend trading if unusual events occur.[3]

FUTURES EXCHANGES

Futures trading takes place on over fifty futures exchanges around the world. Table 8.1 lists the exchanges and their Web addresses.

One advantage of such global futures trading, particularly when it is fully automated, is the potential it offers for linkages between exchanges. For example, the Chicago Mercantile Exchange and the Singapore International Monetary Exchange (SIMEX) are linked so that a trader opening a position in Eurodollars on one exchange can close the position on the other. Linkages among futures exchanges around the world are growing increasingly common.

As previously noted, a trend in futures trading worldwide is the use of electronic trading systems. For a fee, computer terminals are placed in trading rooms of institutions and individuals. These terminals permit traders to submit bids and offers and to consummate trades by simply moving the computer's mouse over the bid or offer the trader wishes to accept and then clicking. Alternative systems match up bids and offers automatically. The Chicago Mercantile Exchange (CME) began the process with a system called GLOBEX, which eventually included access to trading not only in the CME's contracts but also in contracts of certain other exchanges. Several U.S. and several foreign exchanges have developed their own electronic trading systems, and some are merging their systems or working on joint ventures to combine their systems.

According to data provided by *Futures Industry* magazine (March/April 2006 issue), almost 883 million contracts were traded at the Chicago Mercantile Exchange in 2005, the busiest futures exchange in the world. The Chicago Board of Trade had a volume of almost 561 million contracts. EUREX, the combined Swiss-German exchange, traded over 785 million contracts. The EURONEXT.LIFFE traded over 344 million contracts. *Futures Industry* estimates that worldwide futures volume in 2005 was almost 3.9 billion contracts, with a little more than 40 percent of this from the United States.

FUTURES TRADERS

The members of the exchange are individuals who physically go on the exchange floor or, on electronic systems, sit at designated terminals and trade futures contracts. There are several ways to characterize these futures traders.

[3]The 1980 grain embargo against the Soviet Union and the 1987 stock market crash were two such cases.

Table 8.1 Exchanges on Which Futures Trade, November 2005

Argentina

Mercado a Término de Rosarie
http://www.rofex.com.ar

Australia

Sydney Futures Exchange
http://www.sfe.com.au

Austria

Wiener Börse AG
http://www.wbag.at

Belgium

Euronext Brussels
http://www.euronext.com

Brazil

Bolsa de Mercadorias & Futuros
http://www.bmf.com.br

Canada

Montreal Exchange
http://www.m-x.ca
Winnipeg Commodity Exchange
http://www.wce.ca

Chile

Santiago Stock Exchange
http://www.bolsadesantiago.com

China

Dalian Commodity Exchange
http://www.dce.com.cn
Shanghai Futures Exchange
http://www.shfe.com.cn
Zhengzhou Commodity Exchange

Denmark

OMX Exchanges-Copenhagen
http://www.cse.dk

Finland

OMX Exchanges-Helsinki
http://www.hex.com/en

Germany

EUREX
http://www.eurexchange.com

Hong Kong

Hong Kong Exchanges
http://www.hkex.hk

Hungary

Budapest Commodity Exchange
http://www.hex.com

India

National Stock Exchange of India
http://www.nse-india.com

Israel

Tel Aviv Stock Exchange
http://www.tase.co.il

Italy

Borsa Italiana
http://www.borsaitalia.it

Japan

Central Japan Commodity Exchange
http://www.c-com.or.jp
Fukuoka Futures Exchange
http://www.ffe.or.jp
Kansai Commodities Exchange
http://www.kanex.or.jp
Osaka Mercantile Exchange
http://www.osamex.com
Tokyo Commodity Exchange
http://www.tocom.or.jp
Tokyo Grain Exchange
http://www.tge.or.jp
Tokyo International Financial Futures Exchange
http://www.tfx.or.jp
Tokyo Stock Exchange
http://www.tse.co.jp

Korea

Korea Futures Exchange
http://www.kofex.com
Korea Stock Exchange
http://www.kse.or.kr

Malaysia

Malaysia Derivatives Exchange
http://www.mdex.com.my

Mexico

Mexican Derivatives Exchange
http://www.mexder.com.mx

Romania

Romanian Commodity Exchange
http://www.brm.ro
Sibiu Monetary Financial & Commodity Exchange
http://www.bmfms.ro

Singapore

Singapore Commodity Exchange
http://www.sicom.com.sg
Singapore International Monetary Exchange
http://www.sgx.com

South Africa

South African Futures Exchange
http://www.safex.co.za

Spain

Meff Renta Fija
http://www.meff.es
Meff Renta Variable
http://www.meff.com

Sweden

OMX Exchange-Stockholm
http://www.omxgroup.com

Switzerland

EUREX
http://www.eurexchange.com

Taiwan

Taiwan Futures Exchange
http://www.taifex.com.tw

United Kingdom

Intercontinental Exchange
http://www.theice.com
London International Financial Futures Exchange
http://www.euronext.com
London Metal Exchange
http://www.lme.co.uk
OM London Exchange
http://www.omxgroup.com

United States

CBOE Futures Exchange
http://cfe.cboe.com
Chicago Board of Trade
http://www.cbot.com
Chicago Mercantile Exchange
http://www.cme.com
Eurex US
www.eurexus.com
Kansas City Board of Trade
http://www.kcbt.com
Minneapolis Grain Exchange
http://www.mgex.com
New York Board of Trade
http://www.nybot.com
New York Mercantile Exchange
http://www.nymex.com
OneChicago
http://www.onechicago.com
Philadelphia Stock Exchange
http://www.phlx.com

General Classes of Futures Traders

All traders on the futures exchange are either commission brokers or locals.

Commission brokers simply execute transactions for other people. A commission broker can be an independent businessperson who executes trades for individuals or institutions or a representative of a major brokerage firm. In the futures industry, these brokerage firms are called futures commission merchants (FCM). The commission broker simply executes trades for the FCM's customers. Commission brokers make their money by charging a commission for each trade.

Locals are individuals in business for themselves who trade from their own accounts. They attempt to profit by buying contracts at a given price and selling them at a higher price. Their trading provides liquidity for the public. Locals assume the risk and reap the rewards from their skill at futures trading. It has been said that locals represent the purest form of capitalism and entrepreneurship.

Because a futures trader can be a local or an FCM, a conflict occasionally arises between traders' loyalty to themselves and their customers' interests. For example, some traders engage in dual trading, in which they trade for themselves and also trade as brokers for others. Dual trading has become very controversial in recent years. To illustrate the conflict that might arise, suppose that a trader holds a set of orders that includes a large order for a customer. Knowing that the price may move substantially when the customer's order is placed, the trader executes a purchase for his or her own account prior to placing the customer's order. There are a number of other ways in which dual trading can be profitable to the trader at the expense of the customer. For this to occur, however, the trader must act unscrupulously. The exchanges argue that abuses of dual trading are rare. Moreover, they claim that dual trading provides liquidity to the market. Some limitations on dual trading have been enacted.

Classification by Trading Strategy

Futures traders can be further classified by the strategies they employ.

Recall that in Chapter 1 we briefly discussed hedging. A hedger holds a position in the spot market. This might involve owning a commodity, or it may simply mean that the individual plans or is committed to the future purchase or sale of the commodity. Taking a futures contract that is opposite to the position in the spot market reduces the risk. For example, if you hold a portfolio of stocks, you can hedge that portfolio's value by selling a stock index futures contract. If the stocks' prices fall, the portfolio will lose value, but the price of the futures contract is also likely to fall. Because you are short the futures contract, you can repurchase it at a lower price, thus making a profit. The gain from the futures position will at least partially offset the loss on the portfolio.

Hedging is an important activity in any futures or derivatives market. This section has given only a cursory overview of it. Chapter 11 devotes a lot of attention to hedging.

Speculators attempt to profit from guessing the direction of the market. Speculators include locals as well as the thousands of individuals and institutions off the exchange floor. They play an important role in the market by providing the liquidity that makes hedging possible and assuming the risk that hedgers are trying to eliminate. Speculating is discussed in more detail in Chapter 11.

Spreaders use futures spreads to speculate at a low level of risk. Like an option spread, a futures spread involves a long position in one contract and a short position in another. Spreads may be intracommodity or intercommodity. An intracommodity spread is like a time spread in options. The spreader buys a contract with one expiration month and sells an otherwise identical contract with a different expiration month. An intercommodity spread, which normally is not used in options, consists of a long position in a futures contract on one commodity and a short position in a contract on another. In some cases, the two commodities even trade on different exchanges. The rationale for this type of spread rests on a perceived "normal" difference between the prices of the two futures contracts. When the prices move out of line, traders employ intercommodity spreads to take advantage of the expected price realignment.

Futures spreads work much like option time spreads in that the long position in one contract is somewhat offset by the short position in the other. There actually is no real difference between this type of spread and a hedge. For example, suppose the contract is on Treasury bills, the current month is October, and the available futures expirations are December, March, and June. A hedger holds Treasury bills and sells a December contract. A spreader holds a December contract and sells a March contract. Each holds a long position in a spot or nearby futures contract and a short position in a deferred futures contract. Each is attempting to profit from one position while expecting a loss on the other. Neither knows which position will make a profit and which will create a loss.

Arbitrageurs attempt to profit from differences in the prices of otherwise identical spot and futures positions. An analogous type of arbitrage that we already covered is the execution of conversions and reversals to take advantage of option prices that fail to conform to put-call parity. In futures markets there are some important theoretical relationships, which we shall study in Chapters 9 and 10. When prices get out of line with these theoretical predictions, arbitrageurs enter the market and execute trades that bring prices back in line. Because arbitrage is designed to be riskless, it resembles hedging and spreading. In many cases, however, it is difficult to determine whether a given strategy is arbitrage, hedging, or spreading.

Classification by Trading Style

Futures traders can also be classified by the style of trading they practice. There are three distinct trading styles: scalping, day trading, and position trading.

Scalpers attempt to profit from small changes in the contract price. Scalpers seldom hold their positions for more than a few minutes. They trade by using their skill at sensing the market's short-term direction and by buying from the public at the bid price and selling to the public at the ask price. They are constantly alert for large inflows of orders and short-term trends. Because they operate with very low transaction costs, they can profit from small moves in contract prices. The practice of making a large number of quick, small profits is referred to as scalping.

Day traders hold their positions for no longer than the duration of the trading day. Like scalpers, they attempt to profit from short-term market movements; however, they hold their positions much longer than do scalpers. Nonetheless, they are unwilling to assume the risk of adverse news that might occur overnight or on weekends.[4]

Position traders hold their transactions open for much longer periods than do scalpers and day traders. Position traders believe they can make profits by waiting for a major market movement. This may take as much as several weeks or may not come at all.

Scalpers, day traders, and position traders are not mutually exclusive. A speculator may employ any or all of these techniques in transactions.

In addition to those who trade on the floor of the exchange, there are many individuals who trade off the exchange floor and employ some of the same techniques.

Off-Floor Futures Traders

Participants in the futures markets also include thousands of individuals and institutions. Institutions include banks and financial intermediaries, investment banking firms, mutual funds, pension funds, hedge funds, and other corporations. In addition, some farmers and numerous individuals actively trade futures contracts, particularly today, with increasing access and low cost through the Internet.

In addition to those who directly participate in trading, U.S. federal law recognizes and regulates certain other participants. An introducing broker (IB) is an individual who solicits orders from public customers to trade futures contracts. IBs do not execute orders themselves, nor do their firms; rather, they subcontract with FCMs to do this. The IB and the FCM divide the commission.

A commodity trading advisor (CTA) is an individual or firm that analyzes futures markets and issues reports, gives advice, and makes recommendations on the purchase and sale of contracts. CTAs earn fees for their services but do not necessarily trade contracts themselves.

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[4]Day trading in this context should be distinguished from the trading of securities and derivatives on the Internet.

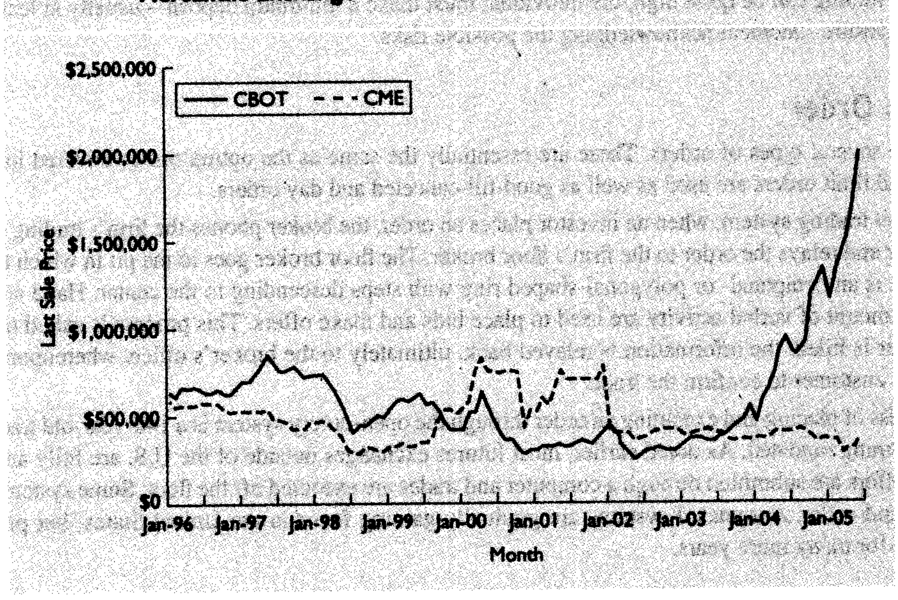
A commodity pool operator (CPO) is an individual or firm that solicits funds from the public, pools them, and uses them to trade futures contracts. The CPO profits by collecting a percentage of the assets in the fund and sometimes through sales commissions. A CPO essentially is the operator of a futures fund, a topic discussed later in this chapter. Some commodity pools are privately operated, however, and are not open for public participation.

An associated person (AP) is an individual associated with any of the above individuals or institutions or any other firm engaged in the futures business. APs include directors, partners, officers, and employees but not clerical personnel.

Costs and Profitability of Exchange Membership

Most futures exchanges have a limited number of full memberships, called seats. There usually is a market for seats, with the highest and lowest bids publicly reported. Seat prices tend to fluctuate with the amount of trading activity in the market and the number of new contracts introduced. Figure 8.1 shows ten recent years of seat prices on two U.S. exchanges, the Chicago Board of Trade and the Chicago Mercantile Exchange. Like stocks, the prices of futures exchange seats fluctuate randomly.

FIGURE 8.1 Monthly Seat Prices on the Chicago Board of Trade and the Chicago Mercantile Exchange



Seats can also be leased at a monthly rate of usually 1.0 to 1.5 percent of the seat price. Some exchanges have different levels of membership. For example, the Chicago Board of Trade has 1,402 full members and a lesser number of Associate Members; Government Instrument Market Membership Interests; Index, Debt, and Energy Membership Interests; and Commodity Options Membership Interests. Associate Members can trade in all markets except agricultural futures. Government Instrument Market Memberships allow trading in futures contracts on government securities. Index, Debt, and Energy Membership Interests allow trading in futures on stock indices, bonds, and any energy-related contracts as well as options on those futures. Commodity option memberships allow trading in any options on futures contracts.

Like options markets, futures markets do not create or destroy wealth. Therefore, one trader's gains are another's losses subject to some slippage due to commissions and taxes. It has been said, however, that the vast majority of futures traders lose money. It is not surprising that a small number of individuals probably earn large profits while assuming high risks. Some are lucky and, of course, some are unlucky.

Forward Market Traders

The forward market is dominated by large institutions, such as banks and corporations. A typical forward market trader is an individual sitting at a desk with a telephone and a computer terminal. Using the computer or telephone, the trader finds out the current prices available in the market. The trader can then agree upon a price with another trader at another firm. The trader may represent his or her own firm or may execute a trade for a client such as a corporation or hedge fund. The trade may be a hedge, a spread, or an arbitrage. In fact, it is the thousands of traders off the floor whose arbitrage activities play a crucial role in making the market so efficient.

It would be remiss to suggest that the forward and futures markets are not linked. In a formal sense, forward contracts cannot be reversed by futures contracts. It is common, however, that a trader will do a forward contract and then immediately do a futures contract to hedge the forward market risk. In fact, the trader might even combine these positions with an option or a swap. Why a trader might do this is a subject we shall get into later when we look at hedging, relationships between the prices in these markets, and risk management.

MECHANICS OF FUTURES TRADING

Before placing an order to trade futures contracts, an individual must open an account with a broker. Because the risk of futures trading can be quite high, the individual must make a minimum deposit—usually at least $5,000—and sign a disclosure statement acknowledging the possible risks.

Placing an Order

One can place several types of orders. These are essentially the same as the option orders covered in Chapter 2. Stop orders and limit orders are used as well as good-till-canceled and day orders.

Under a pit trading system, when an investor places an order, the broker phones the firm's trading desk on the exchange floor and relays the order to the firm's floor broker. The floor broker goes to the pit in which the contract trades. The pit is an octagonal- or polygonal-shaped ring with steps descending to the center. Hand signals and a considerable amount of verbal activity are used to place bids and make offers. This process is called open outcry. When the order is filled, the information is relayed back, ultimately to the broker's office, whereupon the broker telephones the customer to confirm the trade.

The process of placing and executing an order through the open-outcry system is a 140-year-old tradition. That is slowly becoming outdated. As noted earlier, most futures exchanges outside of the U.S. are fully automated so that bids and offers are submitted through a computer and trades are executed off the floor. Some systems will even match buyer and seller. Automated systems are gradually gaining favor in the United States, but pit trading is unlikely to die for many more years.

Role of the Clearinghouse

At this point in the process, the clearinghouse intervenes. Each futures exchange operates its own independent clearinghouse. The clearinghouse in futures markets works like that in options markets, so its basic operations should be familiar to you from Chapter 2.

The concept of a clearinghouse as an intermediary and guarantor to every trade is not nearly as old as the futures markets themselves. The first such clearinghouse was organized in 1925 at the Chicago Board of Trade. The clearinghouse is an independent corporation, and its stockholders are its member clearing firms. Each firm maintains a margin account with the clearinghouse and must meet minimum standards of financial responsibility.

For each transaction, obviously, there is both a buyer, usually called the long, and a seller, typically called the short. In the absence of a clearinghouse, each party would be responsible to the other. If one party defaulted, the other would be left with a worthless claim. The clearinghouse assumes the role of intermediary to each transaction.

It guarantees the buyer that the seller will perform and guarantees the seller that the buyer will perform. The clearinghouse's financial accounts contain separate records of contracts owned and the respective clearing firms and contracts sold and the respective clearing firms. Note that the clearinghouse keeps track only of its member firms. The clearing firms, in turn, monitor the long and short positions of individual traders and firms. All parties to futures transactions must have an account with a clearing firm or with a firm that has an account with a clearing firm.

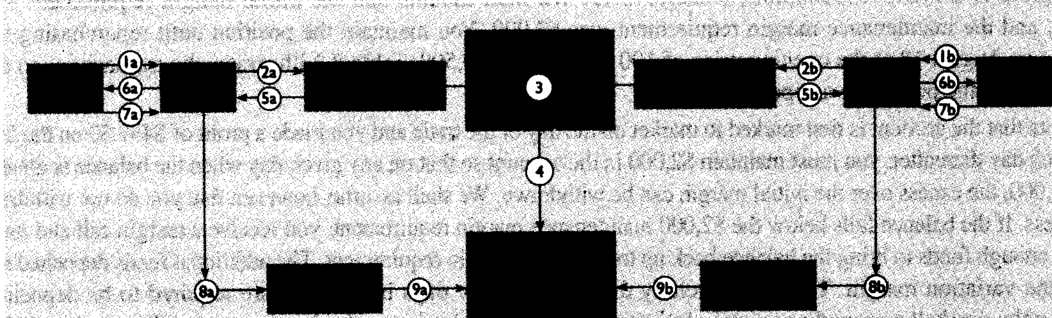
Figure 8.2 illustrates the flow of money and information as a futures transaction is consummated and cleared. Let us illustrate how the clearinghouse operates by assuming that you sell a U.S. Treasury bond futures contract at a price of 97 27/32, which is $97,843.75. You have contacted your broker, who either is an FCM or contracts with an FCM, whose commission broker finds a buyer in the U.S. Treasury bond futures pit of the Chicago Board of Trade. The buyer might be a local, or a commission broker, representing a customer off the floor.

Your brokerage firm clears its trades through ABC Futures, a member firm of The Clearing Corporation, a company that clears trades for the Chicago Board of Trade (TCC). The buyer's FCM clears through ACME Trading Company, a clearing firm that is also a member of TCC. The required margin changes often on these contracts; we shall assume it is $2,500. You deposit this amount with ABC. ABC pools the transactions of all of its customers and deposits an amount required in its account with TCC. The buyer deposits the same amount with ACME, which also deposits a sum of money, based on its customers' open positions, with TCC.

TCC guarantees the performance of you and the buyer. Thus, neither of you has to worry about whether the other will be able to make up the losses. TCC will look to the clearing firms, ABC and ACME, for payment, and they in turn will look to you and the buyer.

Of course the forward market is an over-the-counter market. The parties to the contract deal directly with each other. There is currently no clearinghouse to guarantee to each party that the other will perform, although there are plans under way to begin a type of clearinghouse for certain over-the-counter transactions.

FIGURE 8.2 **A Transaction on the Futures Exchange**



(1a)(1b) Buyer and seller instruct their respective brokers to conduct a futures transaction.
(2a)(2b) Buyer's and seller's brokers request that their firms' commission brokers execute the transaction.
(3) Both commission brokers meet in the pit on the floor of the futures exchange and agree on a price.
(4) Information on the trade is reported to the clearinghouse.
(5a)(5b) Both commission brokers report the price obtained to the buyer's and seller's brokers.
(6a)(6b) Buyer's and seller's brokers report the price obtained to the buyer and seller.
(7a)(7b) Buyer and seller deposit margin with their brokers.
(8a)(8b) Buyer's and seller's brokers deposit margin with their clearing firms.
(9a)(9b) Buyer's and seller's brokers' clearing firms deposit margin with clearinghouse.

Note: Either buyer or seller (or both) could be a floor trader, eliminating the broker and the commission broker.

Daily Settlement

One way in which the clearinghouse helps ensure its survival is by using margins and the daily settlement of accounts. For each contract there is both an initial margin, the amount that must be deposited on the day the transaction is opened, and a maintenance margin, the amount that must be maintained every day thereafter. There are also initial and maintenance margins for spread and hedge transactions, which usually are lower than those for purely speculative positions.

The margin deposit is not quite like the margin on a stock trade. In stock trading, the investor deposits margin money and borrows the remainder of the stock price from the broker. In futures trading, not only is the margin requirement much smaller, but the remainder of the funds are not borrowed. The margin deposit is more like a good-faith security deposit. In fact, some prefer to call them performance bonds rather than margins so that the distinction is clear. In any case, some large and actively trading investors are able to deposit Treasury bills for margins. Others are required to deposit cash.

At the end of each day, a committee composed of clearinghouse officials establishes a settlement price. This usually is an average of the prices of the last few trades of the day. Using the settlement price, each account is marked to market. The difference in the current settlement price and the previous day's settlement price is determined. If the difference is positive because the settlement price increased, the dollar amount is credited to the margin accounts of those holding long positions. Where does the money come from? It is charged to the accounts of those holding short positions. If the difference is negative because the settlement price decreased, the dollar amount is credited to the holders of short positions and charged to those holding long positions.

This process, sometimes called the daily settlement, is an important feature of futures markets and a major difference between futures and forward markets. In forward markets, the gains and losses are normally incurred at the end of the contract's life, when delivery is made. Futures markets credit and charge the price changes on a daily basis. This helps ensure the markets' integrity, because large losses are covered a little at a time rather than all at expiration, by which time the holder of the losing position may be unable to cover the loss.

To illustrate the daily settlement procedure, let us consider the transaction we previously described. We assume that it was initiated on Friday, August 1 of a given year. You sold one Treasury bond futures contract on the Chicago Board of Trade at that day's opening price of 97 27/32. One such contract is for a face value of $100,000, so the price is 0.9784375($100,000) = $97,843.75. We shall assume that the initial margin requirement was $2,500, and the maintenance margin requirement was $2,000. You maintain the position until repurchasing the contract on August 18 at the opening price of 100 16/32, or $100,500. Table 8.2 illustrates the transactions to the account while the position was open.

Note that the account is first marked to market on the day of the trade and you made a profit of $437.50 on the first day. Each day thereafter, you must maintain $2,000 in the account so that on any given day when the balance is greater than $2,000, the excess over the initial margin can be withdrawn. We shall assume, however, that you do not withdraw the excess. If the balance falls below the $2,000 maintenance margin requirement, you receive a margin call and must deposit enough funds to bring the balance back up to the initial margin requirement. The additional funds deposited are called the variation margin. They are officially due within a few days but usually are required to be deposited immediately; we shall assume the money is deposited before trading begins on the day you receive the margin call. By the end of the day on Friday, August 8, the account shows a balance of $1,250. On Monday morning you receive a margin call for $1,250, which is immediately deposited. Another margin call, for $1,937.50, follows the next morning. Undaunted and still confident of ultimately turning things around, you make the deposit on August 12. When you finally buy back the contract on August 18, the account balance withdrawn is $3,031.25. In all, you have made deposits of $2,500, $1,250, and $1,937.50 for a total of $5,687.50. Thus, the overall loss on the trade is $2,656.25.

The preceding example is not at all unusual. It was selected at random. You, the seller of this contract, quickly incurred a substantial loss. For every dollar lost, however, there is a dollar gained by someone else. Traders who were bullish on Treasury bond futures from August 1 to 18 did quite well. Nonetheless, the large dollar flows from day to day serve as a stern reminder of the substantial leverage component that futures contracts offer.

It is also important to note that with futures contracts it is possible to lose more money than one has invested. For example, assume that the market makes a substantial move against the investor. The account balance is depleted, and the broker asks the investor to deposit additional funds. If the investor does not have the funds, the broker will attempt to close out the position. Now assume that the market moves quickly before the contracts can be closed. Ultimately the contracts are sold out, but not before the investor has incurred additional losses. Those losses must be covered in cash. In the limit, a long position can ultimately lose the full price of the contract. This would occur if the price went to zero. On a short position, however, there is no upper limit on the price. Therefore, the loss theoretically is infinite.

One method that futures exchanges use to limit the losses incurred on any given day is the daily price limit, which we briefly discussed earlier. Also the clearinghouse can request that additional margin funds be deposited during a trading session rather than waiting until the end of the day.

Of course in forward markets, there is no clearinghouse, price limits, or daily settlement. There may or may not be margins required; this would depend on the credit-worthiness of the party. In many cases credit is established by posting a letter of credit provided by a bank, which, in effect, stands ready to lend money to cover losses.[5]

The total number of futures contracts outstanding at any one time is called the open interest. The concept is the same as it is for options markets. Each contract has both a long and a short position and counts as one contract of open interest.

Most futures traders do not hold their positions to expiration; rather, they simply reenter the market and execute an offsetting transaction. In other words, if one held a long position in a contract, one might elect to simply sell that contract in the market. The clearinghouse would properly note that the trader's positions were offsetting. If the position were not offset before the expiration month, delivery would become likely.

Delivery and Cash Settlement

All contracts eventually expire. As noted earlier, each contract has a delivery month. The delivery procedure varies among contracts. Some contracts can be delivered on any business day of the delivery month. Others permit delivery only after the contract has traded for the last day—a day that also varies from contract to contract. Still others are cash settled; thus, there is no delivery at all.

Most non-cash-settled financial futures contracts permit delivery any business day of the delivery month. Delivery usually is a three-day sequence beginning two business days prior to the first possible delivery day. The clearing member firms report to the clearinghouse those of their customers who hold long positions. Two business days before the intended delivery day, the holder of a short position who intends to make delivery notifies the clearinghouse of its desire to deliver. This day is called the position day. On the next business day, called the notice of intention day, the exchange selects the holder of the oldest long position to receive delivery. On the third day, the delivery day, delivery takes place and the long pays the short. For most financial futures, delivery is consummated by wire transfer.

Most futures contracts allow for more than one deliverable instrument. The contract usually specifies that the price paid by the long to the short be adjusted to reflect a difference in the quality of the deliverable good. We shall look at this more closely in Chapter 10.

On cash-settled contracts, such as stock index futures, the settlement price on the last trading day is fixed at the closing spot price of the underlying instrument, such as the stock index. All contracts are marked to market on that day, and the positions are deemed to be closed. One exception to this procedure is the Chicago Mercantile Exchange's S&P 500 futures contract, which closes trading on the Thursday before the third Friday of the expiration month but bases the final settlement price on the opening stock price on Friday morning. This procedure was installed to avoid some problems created when a contract settles at the closing prices.

The fact that all futures contracts can be delivered or cash settled is critical to their pricing. About 99 percent of all futures contracts are not delivered or cash settled. Most traders close out their positions prior to expiration, a process called offsetting. The futures market is not the best route to acquiring the underlying asset, because the long

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[5]Some swaps and other over-the-counter contracts periodically settle gains and losses but to date this is the exception and not the rule.

Table 8.2 Daily Settlement Example

Assume that on Friday, August 1, you sell one Chicago Board of Trade September Treasury bond futures contract at the opening price of 97 27/32 ($97,843.75). The initial margin requirement is $2,500, and the maintenance margin requirement is $2,000. You maintain your position every day through Friday, August 15, and then buy back the contract at the opening price on Monday, August 18. The example below illustrates the daily price changes and the flows in and out of the margin account.

Date	Settlement Price	Settlement Price ($)	Mark-to-Market	Other Entries	Account Balance	Explanation
8/1	97-13	$97,406.25	+$437.50	+$2,500	+$2,937.50	Initial margin deposit of $2,500. Price fell by 14/32 leaving profit by day-end of $437.50.
8/4	97-25	97,781.25	−375.00		2,562.50	Price rose by 12/32 giving loss of $375. Balance exceeds maintenance requirement.
8/5	96-18	96,562.50	+1,218.75		3,781.25	Price fell by 1 17/32 giving profit of $1,218.75. Balance well above maintenance requirement.
8/6	96-07	96,218.75	+343.75		4,125.00	Price fell by 11/32 giving profit of $343.75. Balance well above maintenance requirement.
8/7	97-05	97,156.25	−937.50		3,187.50	Price rose by 30/32 giving loss of $937.50. Balance still above maintenance requirement.
8/8	99-03	99,093.75	−1,937.50		1,250.00	Price rose by 1 30/32 giving loss of $1,937.50. Balance short by $750.
8/11	101-01	101,031.25	−1,937.50	+1,250.00	562.50	$1,250.00 deposited. Price rose 1 30/32 giving loss of $1,937.50. Balance short by $1,437.50.
8/12	99-25	99,781.25	+1,250.00	+1,937.50	3,750.00	$1,937.50 deposited. Price fell 1 8/32 giving profit of $1,250. Balance well above maintenance requirement.
8/13	101-01	101,031.25	−1,250.00		2,500.00	Price rose by 1 8/32 giving loss of $1,250. Balance still above maintenance requirement.
8/14	100-25	100,781.25	+250.00		2,750.00	Price fell by 8/32 giving profit of $250. Balance still above maintenance requirement.
8/15	100-25	100,781.25	0.00		2,750.00	Price did not change.
8/18	100-16	100,500.00	+281.25	−3,031.25	0.00	Bought contract back at gain of 9/32 or $281.25. Withdrew remaining balance.

contract holder is at the mercy of the short contract holder. The short can deliver on any of several delivery days and can choose to deliver any of several related but slightly different assets. The long must accept whatever the short offers. Thus, the long usually closes out the position early. Therefore, if one needs the underlying asset, one often will do better to purchase it in the spot market.

Despite the flexibility sellers have in determining the delivery terms, some contracts actually are delivered through a process called exchange for physicals (EFP), also known as against actuals or versus cash. This is, in fact, the only type of permissible futures transaction that occurs off the floor of the exchange, or outside its electronic trading platform, if it has one. In an EFP transaction, the holders of long and short positions get together and agree on a cash transaction that would close out their futures positions. For example, farmer A holds a short position in a wheat contract and firm B holds a long position in the same contract. Firm B wants to buy farmer A's wheat, but either the wheat is not one of the grades acceptable for delivery on the contract or the farmer would find it prohibitively expensive to deliver at Chicago or Toledo as the contract requires. In either case, farmer A and firm B could arrange for A to deliver the wheat at some other acceptable location and B to pay A for the wheat at an agreed-upon price. Then A and B would be permitted to report this transaction to the CBOT as though each had offset its futures contract with a trade with the other party. Thus, the EFP market simply gives the parties additional flexibility in making delivery, choosing the terms, and conducting such business when the exchanges are closed. EFPs can also be used in cash settlement contracts. EFPs are used in several futures markets and compose almost 100 percent of deliveries made in oil futures markets.

Because forward contracts do not have standard terms and conditions like futures contracts, it is not as simple to offset them the way one can futures. If an institution has purchased a forward contract obligating it to purchase a commodity at a future date, however, it can generally sell a new forward contract obligating it to deliver the commodity on that date. Of course, the price negotiated may lead to a profit or a loss. While this type of transaction is basically an offsetting transaction, the liquidity of the forward market may be such that it might be somewhat more difficult to find someone to do the precise transaction the institution wants. In addition, the new transaction also entails some credit risk. In the most liquid forward, offsetting is a fairly simple procedure, as noted above, and is subject to essentially no credit risk. In some cases, however, a forward contract can be offset by entering into the opposite transaction with the same party with which one contracted originally. By mutual agreement, the parties can treat the contracts as offsetting, thereby terminating both contracts and eliminating the credit risk.

When a forward contract is written, the intention is generally to hold it to the expiration day and take delivery. Some contracts, however, do stipulate a cash settlement. Thus, an expiring forward contract works pretty much like an expiring futures contract.

FUTURES PRICE QUOTATIONS

Futures prices are available daily in *The Wall Street Journal* and in many newspapers in large cities. The information provided is usually the open, high, low, and settlement prices, the change in the settlement price, the highest and lowest prices during the life of the contract, and the open interest. The volume is provided for all contracts of the same underlying, but not for each individual contract expiration. Though the print version of *The Wall Street Journal* covers more than just U.S. futures contracts, it includes information only on the most active futures contracts. Information on other contracts is available on the Journal's Web site, http://www.wsj.com.

As with option prices obtained from newspapers, futures prices are also outdated by the time they are available. In addition, the prices are not synchronized with the price of the underlying. For better information without paying a fee, the Internet is the best source. *The Wall Street Journal's* web site provides free quotes on all futures contracts, showing the last trade price, the open, high, low, the volume, and open interest. In addition, the site shows the time of the last transaction. Probably the best sources of price information are the Web sites of the futures exchanges. Although these sites typically do not provide bid and ask prices, they do provide the most current information, such as the last trade, and the volume and open interest figures. This information is nearly always provided with a short delay. Some of the exchanges allow you to download historical data for free, which can include a complete list of every transaction, the price, and the time it took place. Real-time quotes require payment of a fee. There are a few other services on the Internet that provide futures price information, which is obtained from the exchanges, but the information is formatted differently and can be more useful.

See the accompanying "Derivative Tools: Concepts, Applications, and Extensions, Reading Futures Price Quotations" (pp. 257–258) for how to access and interpret this information.

TYPES OF FUTURES CONTRACTS

Many types of futures contracts trade on futures exchanges around the world. Some of these contracts are essentially the same underlying commodity. Many of those listed are not actively traded; some have not been traded at all for some time. Keep in mind that new contracts are frequently introduced and old contracts are sometimes delisted. A brief discussion of some of the characteristics of each major group of contracts follows.

Agricultural Commodities

This category is the oldest group of futures contracts. It includes widely used grains such as wheat, corn, oats, soybeans, and rice. In addition, futures are actively traded on livestock such as cattle and hogs, and various food products such as coffee, cocoa, orange juice, and sugar. Also, futures are traded on cotton, wool, and wood.

Most of the volume of trading in these contracts is in U.S. markets, but there is also some activity in these types of contracts in Canada and a few other countries. Participants in these markets include farmers and firms that use these raw materials as vital inputs.

An important characteristic of these types of futures contracts is that buying and holding these assets typically incurs significant costs. Moreover, the spot markets for these assets are usually quite dispersed. In other words, there is often no central spot market. Hence, it is difficult to refer to a single spot price. A grain harvested in Iowa will be a different product if delivered in Chicago, due to the costs of transporting it and any losses involved when the product is shipped. In addition, the delivery of these assets at expiration of a futures contract will also incur significant costs.

Many of these products are characterized by seasonal production cycles. Hence, the supply of the asset will tend to be augmented at certain times of the year. Shortages and surpluses are common.

Natural Resources

Futures contracts are actively traded on two types of natural resources: metals and energy products. The primary metals on which futures are traded are gold, silver, and copper, with lighter trading on platinum and palladium. Because these products are primarily used in manufacturing, market participants often include companies that buy and sell these products and the products they are made into. Gold and silver, however, have a special status as popular precious metals that people often use as a store of value. Some futures trading in these instruments is based on views about such factors as inflation, international politics, and the world economy, whereby gold and silver are seen as defensive investments against political and economic instability.

Miscellaneous Commodities

This category includes contracts that have mostly traded very lightly and many are no longer listed. They include futures on fertilizer, shrimp, electricity, rubber, glass, cement, potatoes, peanuts, sunflower seeds, inflation, peas, flax, kerosene, yarn, and shipping freight rates. There have also been futures on various measures of weather, including weather-related insurance claims. These futures have not been actively traded, but weather derivatives have found a home in the over-the-counter markets, and we cover this type of product in Chapter 14.

In addition, futures have been traded on indices that reflect the average of the prices of a combination of other futures contracts.

Foreign Currencies

As noted earlier in this chapter, there is a very large forward market in foreign currencies. Futures contracts on foreign currencies are also traded, but these are not as active as are forward contracts. Foreign currency futures, however, have an important historical legacy in that they were the first financial futures contracts. Launched in 1972 in response to a move on the part of nations to allow their currencies to freely float in relation to each other, they were the first futures contracts on money, which some view as the ultimate commodity.

Although there are about fifty currencies with at least a moderately active interbank market, the futures contracts with much volume are those involving the U.S. dollar, the euro, the Japanese yen, the British pound, the

Derivatives Tools

Concepts, Applications, and Extensions
Reading Futures Price Quotations

Current (10-minute delay) futures prices quotations can be obtained from several sources. *The Wall Street Journal's* Web site contains quotes, which can be accessed by choosing (from the menu on the left) "Markets" and then "Commodities." Scroll to the bottom and select the contract group. A pull-down menu will enable you to select the specific contracts. You can also access quotes for options on futures from this menu. The futures quote information is provided by FutureSource. Here we shall take a look at how FutureSource reports the information.

Access the site http://www.futuresource.com. Choose the "Quotes" tab. You can then select a contract, enter a contract symbol, or choose from the list of contracts at the bottom of the page. Suppose that we are interested in a quote for the S&P 500 futures contract. The information below was obtained at 12:45 on the afternoon of November 1, 2005.

Consider the first contract, identified as SPZ05. The SP identifies the Standard and Poor's index. Z is the month. Months follow a pattern of January = F, February = G, . . . , December = Z (the letters I, L, O, P, R, S, T, W, Y are skipped). The 05 represents the year 2005. The second column indicates the month and year of the contract, however, so it is really not necessary to know the symbols. The third column gives the time of the last trade. Note that the third row contains the June 06 contract, which had not yet traded on November 1. It had last traded on October 31 at 15:24:33. The fourth column gives the last trade, which is in bold (y indicates that

Symbol	Month	Time*	Last	Chg	Open	High	Low
SPZ05	Dec '05	13:04:36	**1205.80**	−4.00	1203.90	1209.10	1203.00
SPH06	Mar '06	10:40:48	**1213.50**	−3.70	1215.00	1216.00	1212.80
SPM06	Jun '06	15:24:33	**1225.90y**				
...	...	...	...	...	...	...	...
SPY	N/A	13:04:49	**1203.07**	−3.94	1205.85	1206.04	1201.90

*Exchange local time

the contract has not yet traded this day). The next column gives the change in the last price from the settlement price. The last three columns give the open, high, and low price for the day.

The fourth data row contains ellipses. The actual screen will show all of the available contracts, even if they did not trade, such as the June 06 contract shown above. The final row, however, shows the actual Standard and Poor's 500 index, indicated by the symbol SPY. The spot price is not shown for all contracts, such as corn or Treasury bonds. It is shown only when there is a specific, easily identifiable spot market price.

At the end of the day, the last trade is replaced with the settlement price and the volume is not available.

FutureSource does not provide quotes for options on futures.

The Chicago Mercantile Exchange's Web site can be used to obtain similar information. Go to the site http://www.cme.com. Select "Market Data," "intraday quotes," and "10-min futures and options." Then scroll down to the list of contracts. Consider the quotes below for the S&P 500 futures, which were taken at 12:50 P.M. on November 1, 2005.

The CME's page includes both the last trade and the settlement price, though the latter will not appear until the end of the day. PT CHGE is the point change of the last transaction. The value of −400 means −4.00 points. The estimated volume is shown as about 20,000 contracts for the December 05 contract and 1232 contracts for the March 06 contract. The last three columns give information from the previous trading session. The actual table on the CME's Web site shows all contracts, even those that have not traded. The final two rows indicate the estimated volume for all S&P contracts and the volume and open interest totals for the previous day.

Bid and ask prices are not generally available for free, nor are prices more current than a 10-minute delay.

	SESSION							---- PRIOR DAY ----		
MTH/ STRIKE	OPEN	HIGH	LOW	LAST	SETT	PT CHGE	EST VOL	SETT	VOL	INT
DEC05	1207.50	1208.50	1204.20	887.80	----	−400	20K	1209.80	56188	558747
MAR06	1215.00	1216.20B	1212.20A	888.00A	----	−370	1232	1217.20	1337	55382
JUN06	----	----	1223.90A	889.00B	----	−200	1225	1225.90	1162	4298
...	...	...	...	...	...	...	...	...	...	...
TOTAL						EST. VOL		VOL	OPEN INT.	
TOTAL						22537		58687	646289	

(The A and B indicate ask and bid prices for cases in which a specific transaction did not take place.)

Swiss franc, the Canadian dollar, and the Mexican peso. Even those contracts have almost relatively small volume compared to the forward market for those currencies.

Federal Funds and Eurodollars

Federal funds contracts trade on the Chicago Board of Trade and Eurodollar contracts trade on the International Monetary Market of the Chicago Mercantile Exchange. In the United States, the Eurodollar contract is the most actively traded futures contract. We shall explore these contracts in great detail in later chapters.

Treasury Notes and Bonds

Treasury note and bond contracts, which are traded on the Chicago Board of Trade, are virtually identical except that there are three T-note contracts that are based on 2-year, 5-year, and 10-year maturities while the T-bond contract is based on Treasury bonds with maturities of at least 15 years that are not callable for at least 15 years. Thus, the T-note contracts are intermediate-term interest rate futures contracts and the T-bond contract is a long-term interest rate futures contract. The T-note contracts are traded quite actively, but the T-bond contract is one of the most active of all futures contracts. Other than the difference in maturity of the underlying instruments and the margin requirements, the contract terms are essentially identical. We shall use these contracts extensively in later chapters.

Swap Futures

Swap futures contracts trade on the Chicago Mercantile Exchange. Futures contracts are offered for 2-year, 5-year, and 10-year interest rate swaps. We will cover interest rate swaps in detail in Chapter 12. Presently, swap futures have very low trading volume.

Equities

Stock index futures have been one of the spectacular success stories of the financial markets in recent years. These cash-settled contracts are indices of combinations of stocks. Investors use them to hedge positions in stock, speculate on the direction of the stock market in general, and arbitrage the contracts against comparable combinations of stocks.

Stock index futures contracts are based on indices of common stocks. The most widely traded contract is the S&P 500 futures at the Chicago Mercantile Exchange. The futures price is quoted in the same manner as the index. The futures contract, however, has an implicit multiplier of $250. Thus, if the futures price is 1500, the actual price is 1500($250) = $375,000. At expiration, the settlement price is set at the price of the S&P 500 index and the contract is settled in cash. The expirations are March, June, September, and December. The last trading day is the Thursday before the third Friday of the expiration month.

As described earlier in this chapter, futures on individual stocks have been actively traded in some countries and began trading in the United States on November 8, 2002. These contracts are generally available only on the most actively traded stocks. Specifications for the U.S. contracts call for each futures to cover 100 shares of stock. The contracts are settled at expiration by actual delivery of the shares, though some contracts in other markets are settled in cash.

Somewhat closely related to single stock futures are futures on relatively narrow indices of stocks. In the U.S., a narrow index is one that contains nine or fewer securities, one in which a single component security comprises more than 30 percent of the index, one in which the five highest-weighted component securities comprise more than 60 percent of the index, or one in which the lowest weighted component securities comprising 25 percent of the index have an average daily trading volume of less than $50 million. Many of these indices will have only about five stocks.

In the U.S., single stock futures are traded by several consortia of exchanges. The Chicago Board of Trade, the Chicago Mercantile Exchange, and the Chicago Board Options Exchange have formed a joint venture called OneChicago (http:// www.onechicago. com) for the trading of single stock futures. It is interesting to note the participation of the CBOE in this new futures market. It had formerly been involved only in the trading of options. The NASDAQ Stock Market has formed a joint venture with the London International Financial Futures Exchange (LIFFE) called NASDAQ/LIFFE Markets, which joins the highly successful NASDAQ markets with LIFFE in Europe for the trading of single stock futures. In June 2003, Euronext.liffe (a subsidiary of LIFFE) acquired the NASDAQ Stock Market's portion. OneChicago appears to be the market leader in single stock futures. In May 2006, OneChicago offered markets for over 196 single stock futures contracts. Trading volume, however, remains relatively low. For example, on Friday, June 9, 2006, OneChicago reported the total contract volume for single stock futures was 17,700 contracts and open interest of 955,300 (volume data available at http://www.onechicago.com).

As noted earlier, futures contracts also trade on equity volatility, though these contracts have not been particularly successful.

Managed Funds

Recent years have witnessed the growth of the managed funds industry. *Managed funds* is simply a term that refers to the arrangement by which an investor hires a professional futures trader to conduct transactions on his or her behalf. The futures manager is a commodity trading advisor (CTA), which we discussed earlier. Managed funds can exist in one of four forms: futures funds, private pools, a specialized contract with one or more CTAs, or hedge funds.[6]

Futures funds, sometimes called commodity funds, are essentially mutual funds that pool investors' money and trade futures. Most funds invest only about 20 percent of their money in margin positions. The remainder is kept in interest-earning assets. Futures funds offer the public a way of participating in the futures market with a very low financial commitment. Often, a fund will accept deposits of $1,000 or less. In some cases, the organizers of a fund guarantee that the investor will not lose more than the original investment.

In spite of the apparent attractiveness of funds, they have been quite controversial and with just cause. Their performance has been highly volatile and their costs are quite high, often running to 20 percent of the value of the fund in a given year.

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[6]In addition to futures, these funds also use options and spot market instruments.

Commodity pools, mentioned earlier in the context of the pool operator, or CPO, are private arrangements that operate much like futures funds. The latter, however, are open to the general public while pools normally solicit funds from specific investors. When the fund reaches a certain size, the pool is closed to other investors. Pools generally require at least a $10,000 investment. Pools have suffered some of the same problems as funds, namely inconsistent performance and heavy costs.

An increasingly popular form of managed futures is the private contractual arrangement with one or more CTAs. A typical one would involve a large institutional investor agreeing to allocate a portion of its funds to a group of CTAs. In some cases, the CTAs are supervised by a consultant or introducing broker. Since these arrangements are negotiated between the institutional investor and the introducing broker and the CTAs, the costs are usually significantly lower. Often a rather large number of CTAs, sometimes 20 to 30, is used, leading to diversified and somewhat stable performance. These arrangements are growing in popularity, particularly among pension funds.

Hedge Funds

The hedge fund industry is technically part of the managed funds industry, but hedge funds have become such a large and powerful force in the market that we put them in their own category. A hedge fund is a privately organized pool of money that is invested in literally any financial instruments on any markets of the world. Although hedge funds actively use futures contracts, they also use options, spot instruments such as stocks and bonds, and over-the-counter instruments such as swaps, structured notes, and forward contracts. A hedge fund typically uses a high degree of leverage and takes short positions as willingly as long positions. In addition, a hedge fund frequently borrows heavily. It should be apparent that a hedge fund is a very risky form of investment. These funds normally take in only investors who have large amounts of money and a willingness to take high risks. Also, hedge funds tend to be quite secretive, often registering their legal status in offshore locations, such as the Cayman Islands. Many hedge funds even maintain a great deal of secrecy from their investors, who trust that the fund organizers will invest their money wisely and manage the risk carefully. This has not always been the case.

The term *hedge fund* is probably a misnomer. The general idea of calling this type of company a hedge fund is that the fund would take short positions, which in some sense hedge long positions. That was the original design of a hedge fund. Today, however, these funds are probably as far from the practice of hedging as one can get. That does not mean they are not legitimate investment outlets, since their success and popularity are well established in the financial system. Hedge funds are major providers of liquidity and employ astute traders whose arbitrage transactions help make the market more efficient.

Options on Futures

As described earlier in this chapter, options on futures trade on many futures exchanges. In most cases throughout the world, the most actively traded futures contracts also have options available on the futures contracts. Of course many also have options trading on the underlying asset itself.

TRANSACTION COSTS IN FORWARD AND FUTURES TRADING

In Chapter 2, we discussed the different types of option trading costs that the public and professional traders incur. In this section, we shall do the same for forwards and futures. There is, however, less material available on the trading costs in these markets. One reason is that futures markets have very low trading costs—indeed, that is one of their major advantages. In addition, the costs of trading futures contracts are less documented than the costs of trading options and stocks. Also, forward markets are private and the costs are less publicized; however, forward contracting, being tailored to the specific needs of the parties, can be quite costly.

Commissions

Commissions paid by the public to brokers are assessed on the basis of a dollar charge per contract. The commission is paid at the order's initiation and includes both the opening and closing commissions; that is, a round-trip commission is charged regardless of whether the trader ultimately closes out the contract, makes or takes delivery, or makes a cash settlement. There is no typical commission rate, but, rates of less than $10 for a round-trip are common.

All traders, whether on or off the exchange floor, incur a minimum charge that is paid to the clearing firm and includes the exchange fee and a fee assessed by the National Futures Association, an organization we shall discuss in a later section. These fees are usually less than $2.00 per contract.

In the forward market, transactions are usually conducted directly with dealers so there is typically no commission. There are, however, significant costs associated with processing the paperwork.

Bid-Ask Spread

A second type of trading cost is the bid-ask spread. Chapter 2 explained the concept of the spread for options. Unlike for options and stock markets, however, there is no real market maker. Many floor traders, particularly spreaders and scalpers, quote prices at which they are willing to simultaneously buy at the bid price and sell at the ask price. The bid-ask spread is the cost to the public of liquidity—the ability to buy and sell quickly without a large price concession. Because the spread is not captured and reported electronically, there is little statistical evidence on its size. The spread usually is the value of a minimum price fluctuation, called a tick, but occasionally equals a few more ticks for less liquid markets.

In the forward markets, bid-ask spreads are set by dealers in much the same way as they are on the exchange. These spreads can be quite large, depending on how eager the dealer and its competitors are to make a trade.

Delivery Costs

A futures trader who holds a position to delivery faces the potential for incurring a substantial delivery cost. In the case of most financial instruments, this cost is rather small. For commodities, however, it is necessary to arrange for the commodity's physical transportation, delivery, and storage. Although the proverbial story of the careless futures trader who woke up to find thousands of pounds of pork bellies dumped on the front lawn certainly is an exaggeration, anyone holding a long position in the delivery month must be aware of the delivery possibility. This no doubt explains part of the popularity of cash settlement contracts.

In forward markets, transactions are tailored to the needs of the parties. Consequently, the terms are usually set to keep delivery costs at a minimum. Cash settlement is frequently used.

REGULATION OF FUTURES AND FORWARD MARKETS

Throughout history, some regulators and legislators have taken a dim view of futures trading, likening it to gambling. In the nineteenth century, there were numerous attempts to outlaw futures trading. In virtually all countries, futures markets are permitted but heavily regulated at the federal level.

In the United Kingdom, the regulatory agency is the Financial Services Authority. In Canada, futures regulation is done at the provincial level so there are several provincial regulatory agencies. In Japan the regulatory agency is called the Financial Services Agency. Similar agencies exist in other countries.

The structure of regulation in the United States is somewhat different from the structure in other countries. As noted in Chapter 2, the financial markets are regulated by several different agencies. The Securities and Exchange Commission, or SEC, (http://www.sec.gov) regulates the securities markets. With the introduction of single stock futures, the SEC regulates a small segment of the futures markets. The CFTC (http://www.cftc.gov) is a completely separate regulatory body that is the primary regulator of futures markets in the United States.

The objective of most federal regulatory agencies is to authorize futures exchanges to operate, approve new contracts and modifications of existing contracts, ensure that price information is made available to the public, authorize individuals to provide services related to futures trading, and oversee the markets to prevent manipulation. In carrying out their mandate, they monitor the market by observing prices, requiring reportable positions, which document large size positions held by traders, and establishing position limits, which are restrictions on the number of contracts a given trader can hold. In authorizing new contracts, the regulatory agencies evaluate proposals to determine if the contracts serve an "economic purpose," which is generally considered to be whether the contract can be used for hedging.

One important role of government in the futures industry is in adjudicating disputes. For example, a customer may feel that he has a legal claim against a broker. Ordinarily the customer would file a lawsuit, but this action is costly for all parties. In most countries, the federal regulatory agency is generally authorized to provide a means of settling such disputes. In the U.S., these disputes are handled within a professional trade organization called the National Futures Association or NFA (http://www.nfa.org). The NFA is authorized to license personnel who will engage in services related to futures markets, monitor and regulate trading rule violations, and impose fines and sanctions. Perhaps its most important role is to provide a means of settling disputes outside of the courtroom. Similar organizations exist in some other countries. Groups like the NFA are referred to as industry self-regulatory organizations.

Banking regulators also indirectly regulate the futures markets by virtue of their regulatory authority over the banking industry, which is a major participant in the futures markets. In addition, regional governments, such as the states in the U.S., have some regulatory authority. Various professional organizations and trade associations also impose standards on their members.

The over-the-counter market for forward contracts, as well as swaps and options, is not regulated directly. Most of the participating institutions are subject to some form of regulation based on their other activities, such as banking and securities. Hence banking and securities regulators do exert some indirect regulation over the over-the-counter derivative markets. Of course, forward market transactions are always subject to ordinary commercial and criminal laws.

Futures markets participants have often complained that the heavy regulation they are subject to and the light regulation that forward markets are subject to has led to an "uneven playing field." Futures markets and forward markets are indeed competitors, offering similar transactions, but futures markets are guaranteed against credit losses and are more open to the public. Forward markets are essentially private transactions. Hence, most governments have taken the view that futures markets must be regulated for the sake of the general public, while forward markets are private transactions that any parties should be able to engage in, provided that they do not violate any other laws.

Questions and Problems

1. What are the differences among scalpers, day traders, and position traders?
2. The crude oil futures contract on the New York Mercantile Exchange covers 1,000 barrels of crude oil. The contract is quoted in dollars and cents per barrel, e.g., $27.42, and the minimum price change is $0.01. The initial margin requirement is $3,375 and the maintenance margin requirement is $2,500. Suppose that you bought a contract at $27.42, putting up the initial margin. At what price would you get a margin call?

3. Suppose that you buy a stock index futures contract at the opening price of 452.25 on July 1. The multiplier on the contract is 500, so the price is \$500 (452.25) = \$226,125. You hold the position open until selling it on July 16 at the opening price of 435.50. The initial margin requirement is \$9,000, and the maintenance margin requirement is \$6,000. Assume that you deposit the initial margin and do not withdraw the excess on any given day. Construct a table showing the charges and credits to the margin account. The daily prices on the intervening days are as follows:

Day	Settlement Price
7/1	453.95
7/2	454.50
7/3	452.00
7/7	443.55
7/8	441.65
7/9	442.85
7/10	444.15
7/11	442.25
7/14	438.30
7/15	435.05
7/16	435.50

4. Compare and contrast cash settlement with physical settlement.
5. Explain the difference between hedge funds and futures funds.
6. What are daily price limits, and why are they used?
7. What are circuit breakers? What are their advantages and disadvantages?
8. Explain the differences among the three means of terminating a futures contract: an offsetting trade, cash settlement, and delivery. How is a forward contract terminated?
9. Compare and contrast three types of futures trading costs.
10. What are the objectives of federal regulation of future markets?
11. What is the objective of an industry self-regulatory organization?
12. How are spread and arbitrage strategies forms of speculation? How can they be interpreted as hedges?
13. What factors would determine whether a particular strategy is a hedge or a speculative strategy?
14. How do locals differ from commission brokers? How do the latter differ from futures commission merchants?
15. List and briefly explain the important contributions provided by futures exchanges.
16. Explain the basic differences between open-outcry and electronic trading systems.
17. **Explain the difference between a forward contract and an option.**
18. What factors distinguish a forward contract from a futures contract? What do forward and futures contracts have in common? What advantages does each have over the other?
19. The open interest in a futures contract changes from day to day. Suppose that investors holding long positions are divided into two groups: A is an individual investor and OL represents other investors. Investors holding short positions are denoted as S. Currently A holds 1,000 contracts and OL holds 4,200; thus, S is short 5,200 contracts. Determine the holdings of A, OL, and S after each of the following transactions.

   a. A sells 500 contracts, OL buys 500 contracts.

b. A buys 700 contracts, OL sells 700 contracts.
c. A buys 200 contracts, S sells 200 contracts.
d. A sells 800 contracts, S buys 800 contracts.

What determines whether volume increases or decreases open interest?

20. How do options on futures differ from options on the asset underlying the futures?
21. What are the various ways in which an individual may obtain the right to go on to the floor of an exchange and trade futures?
22. Explain how the clearinghouse operates to protect the futures market.

Appendix 8

Taxation of Futures Transactions in the United States

Investors' and traders' profits from most futures contracts, as well as index options, are considered to be 60 percent capital gains and 40 percent ordinary income. Capital gains are taxed at the ordinary income rate, but subject to a maximum of 20 percent. Thus, an investor in the 31 percent tax bracket would have futures profits taxed at a blended rate of $0.6(0.20) + 0.4(0.31) = 0.244$.

In addition, all futures and index options profits are subject to a mark to market rule in which accumulated profits are taxable in the current year even if the contract has not been closed out. For example, assume that you bought a futures contract on October 15 at a price of $1,000. Your account was, of course, marked to market daily. At the end of the year, the accumulated profit in the account was $400, meaning that the futures price at the end of the year was $1,400. Then you would have to pay the tax that year on $400 even though you had not closed out the contract. In other words, realized and unrealized profits are taxed and losses are recognized.

These rules apply only to speculative transactions. Taxation of hedge transactions is more complex and will be examined in Chapter 11.

Consider the following example. Suppose that an investor in the 31 percent tax bracket purchases a futures contract at $1,000 on October 15 and ultimately sells it at $1,300 on January 20 of the next year. Assume that the contract price was $1,400 at the end of the year. The first year the tax liability is on $400, so the tax is $\$400(0.6)(0.20) + \$400(0.40)(0.31) = \$97.60$, an effective rate of 24.4 percent. In the second year, there is a taxable loss of $\$1,400 - \$1,300 = \$100$. This can be used to offset taxable gains; thus, it will save the trader $0.244(\$100) = \$24.40$ in taxes on profitable futures trades in that year. Losses can be used to offset gains, but not more than the total amount of taxable gains. Any losses not used can be carried back to offset prior trading profits for up to three years.

Suppose that the contract expired in February and the investor took delivery of the commodity. Let the price at expiration be $1,500. Then, it would be assumed that the commodity was purchased at $1,500. The investor would have paid tax on the $400 profit at the end of the year in which the contract was bought; the investor would owe tax on the $100 profit that accrued between the end of the year and the expiration.

The new futures contracts on individual stocks will be taxed the same way as individual stocks. Gains and losses from offsetting a position can be either long-term or short-term, but most will be short-term because of the short lives of most contracts. If the investor takes delivery of the stock, the futures holding period is added to the stock holding period, which is generally beneficial to the investor.

Although recent tax laws have greatly simplified the taxation of futures contracts, many complexities remain. Competent tax advice is necessary to keep up with the many changes and ensure compliance with the various rules.

Questions and Problems

1. On October 1, you purchase one March stock index futures contract at the opening price of 410.30. The contract multiplier is $500, so the price of 410.30 is really $500(410.30) = $205,150. You hold the position open until February 20, whereupon you sell the contract at the opening price of 427.30. The settlement price on December 31 was 422.40. You are in the 31 percent tax bracket. Compute your tax liability.
2. In November you buy a futures contract on a commodity at a price of $10,000. At the end of the year, the futures price is $10,500. You hold your position open until January 20, at which time the commodity price is $11,200, the contract expires, and you take delivery. You are in the 31 percent tax bracket. Compute your tax liability in both years.

9

PRINCIPLES OF PRICING FORWARDS, FUTURES, AND OPTIONS ON FUTURES

We are now ready to move directly into the pricing of forward and futures contracts. The very nature of the word *futures* suggests that futures prices concern prices in the future. Likewise, the notion of a forward price suggests looking ahead to a later date. But as we shall learn, futures and forward prices are not definitive statements of prices in the future. In fact, they are not even necessarily predictions of the future. But they are important pieces of information about the current state of a market, and futures and forward contracts are powerful tools for managing risk. In this chapter, we shall see how futures prices, forward prices, spot prices, expectations, and the costs of holding positions in the asset are interrelated. As with options, our objective is to link the price of the futures or forward contract to the price of the underlying instrument and to identify factors that influence the relationship between these prices.

In Chapter 1 we noted that there are options in which the underlying is a futures. When we covered options in which the underlying is an asset, we could not cover options on futures because we had not yet covered futures. Because this chapter covers the pricing of futures contracts, we can also cover the pricing of options on futures, as we do later in this chapter.

In the early part of this chapter, we shall treat forward and futures contracts as though they were entirely separate instruments. Recall that a forward contract is an agreement between two parties to exchange an asset for a fixed price at a future date. No money changes hands, and the agreement is binding. In order to reverse the transaction, it is necessary to find someone willing to take the opposite side of a new, offsetting forward contract calling for delivery of the asset at the same time as the original contract. A forward contract is created in the over-the-counter market and is subject to default risk. A futures contract is also an agreement between two parties to exchange an asset for a fixed price at a future date. The agreement is made on a futures exchange, however, and is regulated by that exchange. The contract requires that the parties make margin deposits, and their accounts are marked to market every day. The contracts are standardized and can be bought and sold during regular trading hours. These differences between forward and futures contracts, particularly the marking to market, create some differences in their prices and values. As we shall see later, these differences may prove quite minor; for now, we shall proceed as though forward and futures contracts were entirely different instruments.

GENERIC CARRY ARBITRAGE

In this section our goal is to illustrate the basic principles of pricing forward and futures contracts without reference to any specific type of contract. Unique contract characteristics lead to complexities that are best deferred until the fundamental principles of pricing are understood. Thus, in this section the underlying asset is not identified. It is simply a generic asset.

Concept of Price versus Value

In Chapter 1, we discussed how an efficient market means that the price of an asset equals its true economic value. The holder of an asset has money tied up in the asset. If the holder is willing to retain the asset, the asset must have a value at least equal to its price. If the asset's value were less than its price, the owner would sell it. The value is the present value of the future cash flows, with the discount rate reflecting the opportunity cost of money and a premium for the risk assumed.

Although this line of reasoning is sound in securities markets, it can get one into trouble in forward and futures markets. A forward or futures contract is not an asset. You can buy a futures contract, but do you actually pay for it? A futures contract requires a small margin deposit, but is this really the price? You can buy 100 shares of a $20 stock by placing $1,000 in a margin account and borrowing $1,000 from a broker. Does that make the stock worth $10 per share? Certainly not. The stock is worth $20 per share: You have $10 per share invested and $10 per share borrowed.

The margin requirement on a futures contract is not really a margin in the same sense as the margin on a stock. You might deposit 3 to 5 percent of the price of the futures contract in a margin account, but you do not borrow the remainder. The margin is only a type of security deposit. Thus, the buyer of a futures contract does not actually "pay" for it, and, of course, the seller really receives no money for it. As long as the price does not change, neither party can execute an offsetting trade that would generate a profit. As noted previously, a forward contract may or may not require a margin deposit or some type of credit enhancement, but if it does, the principle is still the same: The forward price is not the margin.

When dealing with forward and futures contracts, we must be careful to distinguish between the forward or futures price and the forward or futures value. The price is an observable number. The value is less obvious. But fortunately the value of a forward or futures contract at the start is easy to determine. That value is simply zero. This is because neither party pays anything and neither party receives anything of monetary value. That does not imply, however, that neither party will pay or receive money at a later date. The values of futures and forward contracts during their lives, however, are not necessarily equal either to each other or to zero.

The confusion over price and value could perhaps be avoided if we thought of the forward or futures price as a concept more akin to the exercise price of an option. We know that the exercise price does not equal an option's value. It simply represents the figure that the two parties agreed would be the price paid by the call buyer or received by the put buyer if the option is ultimately exercised. In a similar sense, the futures or forward price is simply the figure that the two parties have agreed will be paid by the buyer to the seller at expiration in exchange for the underlying asset. This price is sometimes called the "delivery price." While we could call the forward price or the futures price the "exercise price" of the contract, the use of the terms "forward price" and "futures price" or "delivery price" is so traditional that it would be unwise not to use them.

Let us now proceed to understand how the values and prices of forward and futures contracts are determined. First, we need some notation. We let $V_t(0,T)$ and $v_t(T)$ represent the values of forward and futures contracts at time t that were created at time 0 and expire at time T. Similarly, $F(0,T)$ and $f_t(T)$ are the prices at time t of forward and futures contracts created at time 0 that expire at time T. Since a forward price is fixed at a given time, conditional on the expiration date, the price does not change and, therefore, does not require a time subscript. Also, since a futures price does change, it does not matter when the contract was established.

Value of a Forward Contract

Given our statement above that the value of each contract is zero when established, we can initially say that $V_0(0,T) = 0$ and $v_0(T) = 0$.

Forward Price at Expiration The first and most important principle is that the price of a forward contract that is created at expiration must be the spot price. Such a contract will call for delivery, an instant later, of the asset. Thus, the contract is equivalent to a spot transaction, and its price must, therefore, equal the spot price. Thus, we can say

$$F(T,T) = S_T.$$

If this statement were not true, it would be possible to make an immediate arbitrage profit by either buying the asset and selling an expiring forward contract or selling the asset and buying an expiring forward contract.

Value of a Forward Contract at Expiration At expiration, the value of a forward contract is easily found. Ignoring delivery costs, the value of a forward contract at expiration, $V_T(0,T)$, is the profit on the forward contract. The profit is the spot price minus the original forward price. Thus,

$$V_T(0,T) = S_T - F(0,T).$$

When you enter into a long forward contract with a price of F(0,T), you agree to buy the asset at T, paying the price F(0,T). Thus, your profit will be $S_T - F(0,T)$. This is the value of owning the forward contract. At the time the contract was written, the contract had zero value. At expiration, however, anyone owning a contract permitting him or her to buy an asset worth S_T by paying a price F(0,T) has a guaranteed profit of $S_T - F(0,T)$. Thus, the contract has a value of $S_T - F(0,T)$. Of course, this value can be either positive or negative. The value to the holder of the short position is simply minus one times the value to the holder of the long position.

Value of a Forward Contract Prior to Expiration Before we begin, let us take note of why it is important to place a value on the forward contract. If a firm enters into a forward contract, the contract does not initially appear on the balance sheet. Although it may appear in a footnote, the contract is not an asset or liability, so there is no place to put it on the balance sheet. During the life of the forward contract, however, value can be created or destroyed as a result of changing market conditions. For example, we already saw that the forward contract has a value at expiration of $S_T - F(0,T)$, which can be positive or negative. To give a fair assessment of the assets and liabilities of the company, it is important to determine the value of the contract before expiration. If that value is positive, the contract can be properly viewed and recorded as an asset; if it is negative, the contract should be viewed and recorded as a liability. Investors should be informed about the values of forward contracts and, indeed, all derivatives so that they can make informed decisions about the impact of derivative transactions on the overall value of the firm.

Table 9.1 illustrates how we determine the value. As we did in valuing options, we construct two portfolios that obtain the same value at expiration. Portfolio A is a forward contract constructed at time 0 at the price F(0,T). It will pay off $S_T - F(0,T)$ at expiration, time T. To construct Portfolio B, we do nothing at time 0. At time t, we know that the spot price is S_t and that a forward contract that was established at time 0 for delivery of the asset at T was created at a price of F(0,T). We buy the asset and borrow the present value

Table 9.1 Valuing a Forward Contract Prior to Expiration

Portfolio	Composition	Value at 0	Value at t	Value at T
A	Long forward contract established at t at price of F(0,T)	0	$V_t(0,T)$	$S_T - F(0,T)$
B	Long position in asset and loan of $F(0,T)(1 + r)^{-(T-t)}$ established at t	N/A	$S_t - F(0,T)(1 + r)^{-(T-t)}$	$S_T - F(0,T)$

Conclusion: The value of portfolio A at t must equal the value of portfolio B at t.

$$V_t(0,T) = S_t - F(0,T)(1 + r)^{-(T-t)}$$

of F(0,T), with the loan to be paid back at T. Thus, the value of our position is $S_t - F(0,T)(1+r)^{-(T-t)}$. At T, we sell the asset for S_T and pay back the loan amount, F(0,T). Thus, the total value at T is $S_T - F(0,T)$. This is the same as the value of Portfolio A, which is the forward contract. Thus, the value of Portfolio B at t must equal the value of the forward contract, Portfolio A, at t. Hence,

$$V_t(0,T) = S_t - F(0,T)\,(1+r)^{-(T-t)}$$

It is intuitive and easy to see why this is the value of the forward contract at t. If you enter into the contract at time 0, when you get to time t, you have a position that will require you to pay F(0,T) at time T and will entitle you to receive the value of the asset at T. The present value of your obligation is $F(0,T)(1+r)^{-(T-t)}$. The present value of your claim is the present value of the asset, which is its current price of S_t.

Numerical Example Suppose that you buy a forward contract today at a price of $100. The contract expires in 45 days. The risk-free rate is 10 percent. The forward contract is an agreement to buy the asset at $100 in 45 days. Now, 20 days later, the spot price of the asset is $102. The value of the forward contract with 25 days remaining is then

$$102 - 100(1.10)^{-25/365} = 2.65.$$

In other words, at time T we are obligated to pay $100 in 25 days but we shall receive the asset, which has a current value of $102.

Price of a Forward Contract

In this section we consider the initial price of a forward contract. As noted previously, we use the notation F(0,T) for the forward price. We have already noted that the value of a forward contract when originally written is zero; hence we can set the forward contract value equation at time 0 equal to 0,

$$V_0(0,T) = S_0 - F(0,T)(1+r)^{-T} = 0.$$

Solving for the forward price, we have

$$F(0,T) = S_0(1+r)^T.$$

Therefore, the price of a forward contract on a generic asset is simply the future value of the current spot price of the asset, where the future value is obtained by grossing up the spot price by the risk-free interest rate. The forward price is seen as the price that forces the contract value to equal zero at the start. This valuation method is known as the carry arbitrage model or cost of carry model because the forward value depends only on the carrying costs related to the underlying asset. In this case, the forward price depends on the finance carrying costs. In subsequent sections, we will examine unique aspects of forward pricing for various different forward contracts, such as stock indices, currencies and commodities.

Value of a Futures Contract

In this section we shall consider the valuation of futures contracts. As noted previously, we shall use $f_t(T)$ for the futures price and $v_t(T)$ for the value of a futures contract. Let us recall that a futures contract is marked to market each day. We have already established that the value of a futures contract when originally written is zero.

Futures Price at Expiration At the instant at which a futures contract is expiring, its price must be the spot price. In other words, if you enter into a long futures contract that will expire an instant later, you have agreed to buy the asset an instant later, paying the futures price. This is the same as a spot transaction. Thus,

$$f_T(T) = S_T.$$

If this statement were not true, buying the spot and selling the futures or selling the spot and buying the futures would generate an arbitrage profit.

Value of a Futures Contract during the Trading Day but before Being Marked to Market When we looked at forward contracts, the second result we obtained was the value of a forward contract at expiration. In the case of futures contracts, it is more useful to look next at how one values a futures contract before it is marked to market. In other words, what is a futures contract worth during the trading day?

Suppose that we arbitrarily let the time period between settlements be one day. Suppose you purchase a futures contract at $t - 1$ when the futures price is $f_{t-1}(T)$. Let us assume that this is the opening price of the day and that it equals the settlement price the previous day. Now let us say we are at the end of the day, but the market is not yet closed. The price is $f_t(T)$. What is the value of the contract? If you sell the contract, it generates a gain of $f_t(T) - f_{t-1}(T)$. Thus, we can say that the value of the futures contract is

$$v_t(T) = f_t(T) - f_{t-1}(T) \text{ before the contract is marked to market.}$$

The value of the futures contract is simply the price change since the time the contract was opened or, if it was opened on a previous day, the last price change since marking to market. Note, of course, that the value could be negative. If we were considering the value of the futures to the holder of the short position, we would simply change the sign.

Value of a Futures Contract Immediately After Being Marked to Market When a futures contract is marked to market, the price change since the last marking to market or, if the contract was opened during the day, the price change since it was opened is distributed to the party in whose favor the price moved and charged to the party whom the price moved against. This, of course, is the mark to market procedure. As soon as the contract is marked to market, the value of the contract reverts to zero. Thus,

$$v_t(T) = 0 \text{ as soon as the contract is marked to market.}$$

If the futures price was still at the last settlement price and the futures trader then tried to sell the contract to capture its value, it would generate no profit, which is consistent with its zero value.

Thus, to summarize these two results, we find that the value of a long futures contract at any point in time is the profit that would be generated if the contract were sold. Because of the daily marking to market, the value of a futures contract reverts back to zero as soon as it is marked to market. The value for the holder of a short futures contract is minus one times the value for the holder of the long futures contract. For a long futures contract, value is created by positive price changes; for a short futures contract, value is created by negative price changes.

Price of a Futures Contract

In this section we consider the initial price of a futures contract. As noted previously, we use $f_t(T)$ for the futures price. We have already noted that the value of a futures contract when originally written is zero. Assuming the mark to market feature of futures contracts does not impact its current price, then

$$f_t(T) = F(0,T) = S_0(1 + r)^T.$$

Therefore, the price of a generic futures contract is the same as that of a generic forward contract. It is important to note, however, that this result assumes no marking to market. In the next section, we explore the implications of marking to market on pricing futures contracts.

DERIVATIVES TOOLS

Concepts, Applications, and Extensions

When Forward and Futures Contracts are the Same

Assuming no possibility of default, there are several conditions under which forward and futures contracts produce the same results at expiration and, therefore, would have the same prices. First, recall that forward contracts settle their payoffs at expiration. Given the price of the underlying at expiration of S_T and the price entered into when the contract is established, the holder of a long position would have a payoff of

$$S_T - F(0,T).$$

This, as we noted, is the value of the forward contract at expiration. A futures contract is written at a price that changes every day. Thus, if a futures expiring at time T is established at time 0 at a price of $f_0(T)$, the price at the end of the next day will be $f_1(T)$. The following day the price is $f_2(T)$, and this continues until it settles at expiration at $f_T(T)$, which is the spot price, S_T. The last mark to market profit is $f_T(T) - f_{T-1}(T)$. We see that these contracts clearly have different cash flow patterns as summarized below:

Day	Futures Cash Flow	Forward Cash Flow
0	0	0
1	$f_1(T) - f_0(T)$	0
2	$f_2(T) - f_1(T)$	0
3	$f_3(T) - f_2(T)$	0
...		
T−2	$f_{T-2}(T) - f_{T-3}(T)$	0
T−1	$f_{T-1}(T) - f_{T-2}(T)$	0
T	$f_T(T) - f_{T-1}(T) = S_T - f_{T-1}(T)$	$S_T - F(0,T)$

We want to know whether the original futures price, $f_0(T)$, would equal the original forward price, F(0,T). Thus, we now look at the conditions under which they will be equal.

The futures price will equal the forward price one day prior to expiration. This should be obvious. Look at the table of cash flows for futures and forward contracts created one day prior to expiration:

Day	Futures Cash Flow	Forward Cash Flow
T−1	0	0
T	$f_T(T) - f_{T-1}(T) = S_T - f_{T-1}(T)$	$S_T - F(T-1,T)$

The futures price, $f_{T-1}(T)$, would have to equal the forward price, F(T − 1,T), because neither contract requires an outlay at the start, day T−1; both contracts require the payment of an amount of cash ($f_{T-1}(T)$ for the futures and F(T − 1, T) for the forward, at time T; and both contracts produce the amount, S_T, at T. These amounts paid at T, would have to be the same or one could sell the contract requiring the higher payment and buy the contract requiring the lower payment to generate a sure positive payoff without paying anything.

The futures price will equal the forward price two days (or more) prior to expiration if the interest rate one day ahead is known in advance. Suppose that we initiate futures and forward contracts at the end of day T − 2. We hold the position through the end of day T − 1, and then to the end of day T. Let r_1 be the interest rate one day prior to expiration, which is assumed to be known two days prior to expiration. We assume that these are daily rates, so to obtain one day's interest, we just multiply by 1 + r, that is without using an exponent. Let us do the following transactions two days prior to expiration:

Go long one forward contract at the price $F_{T-2}(T)$.

Sell $1/(1 + r_1)$ futures contracts at the price $f_{T-2}(T)$.

Now move forward to the end of day T−1:

The forward contract will have no cash flow.
Buy back the futures for a gain or loss of $-(f_{T-1}(T) - f_{T-2}(T))$. Multiplying by the number of contracts gives an amount of $-[1/(1 + r_1)][f_{T-1}(T) - f_{T-2}(T)]$. Compound this value forward for one day, which means reinvesting at r_1 if this is a gain or financing at r_1 if this is a loss. Then sell one new futures at a price of $f_{T-1}(T)$.

Now, at expiration, we have the following results:

The forward contract will pay off $S_T - F(0,T)$.

The value of the previous day's gain or loss reinvested for one day is

$$-[1/(1 + r_1)][f_{T-1}(T) - f_{T-2}(T)][1 + r_1] = -(f_{T-1}(T) - f_{T-2}(T)).$$

The mark to market profit or loss from the single futures contract is

$$-(f_T(T) - f_{T-1}(T)) = -(S_T - f_{T-1}(T)).$$

The total is

$$S_T - F(0,T) - (f_{T-1}(T) - f_{T-2}(T)) - (S_T - f_{T-1}(T)) = f_{T-2}(T) - F(0,T).$$

When the contracts were first established, these two prices were known, because they were the prices at which the contracts were entered. Thus, this strategy will produce a known amount at expiration. Since there were no initial cash flows, this cash flow at expiration has to be zero. Otherwise, one could sell the higher-priced contract and buy the lower-priced contract. This would require no cash outlay at the start but would produce a positive cash flow at expiration. Thus, the futures price would have to equal the forward price.

If the interest rate one day ahead is not known, this strategy will not be feasible. In that case, the correlation between futures prices and interest rates can tell us which price will be higher, though it will not tell us by how much one price will exceed the other. This point is discussed in this chapter.

Forward versus Futures Prices

At expiration, forward and futures prices equal the spot price, but there are also a few other conditions under which they are equal. First, however, let us assume that there is no default risk. Now consider the case of one day prior to expiration. A futures contract that has only one day remaining will be marked to market the next day, which is at the expiration. The forward contract will be settled at expiration. Thus, the forward and futures contracts have the same cash flows and are, effectively, the same contract.

If we back up two days prior to expiration, the comparison is more difficult. Suppose that we make the assumption that the risk-free interest rate is either the same on both days or that we know one day what the rate will be the next day. Hence, we effectively rule out any interest rate uncertainty. In that case, it can be shown that the forward price will equal the futures price.

If we do not assume interest rate certainty, we can argue heuristically which price will be higher. If interest rates are positively correlated with futures prices, an investor holding a long position will prefer futures contracts over forward contracts, because futures contracts will generate mark to market profits during periods of rising interest rates and incur mark to market losses during periods of falling interest rates. This means that gains will be reinvested at higher rates and losses will be incurred when the opportunity cost is falling. Futures contracts would, therefore, carry higher prices than forward contracts. If interest rates are negatively correlated with futures prices, an investor holding a long position will prefer forward contracts over futures contracts, because the marking to market of futures contracts will be disadvantageous. Then forward contracts would carry higher prices. If interest rates and futures prices are uncorrelated, forward and futures contracts will have the same prices.

Of course, as we have previously noted, forward contracts are subject to default and futures contracts are guaranteed against default by the clearinghouse. Default risk can also affect the difference between forward and futures prices. It would seem that if forward contract buyers (sellers) have greater risk than forward contract sellers (buyers), the forward price would be pushed up (down). The forward market, however, does not typically incorporate credit risk into the price. As we shall cover in Chapter 15, virtually all qualifying participants in over-the-counter markets pay/receive the same price. Parties with greater credit risk pay in the form of collateral or other credit-enhancing measures. Hence, we are not likely to observe differences in forward and futures prices due to credit issues.

By not observing any notable differences in forward and futures prices, we can reasonably assume that forward prices are the same as futures prices. Thus, the remaining material in this chapter, while generally expressed in terms of futures prices, will also apply quite reasonably to forward prices.

CARRY ARBITRAGE WHEN UNDERLYING GENERATES CASH FLOWS

Until now we have avoided any consideration of how intermediate cash flows, such as interest and dividends, affect forward and futures prices. We did note earlier in this chapter that these cash payments would have an effect on the cost of carry, possibly making it negative. Now we shall look more closely at how they affect forward and futures prices. The examples will be developed in the context of futures contracts. Note also that, in this section, we are no longer focusing on a generic asset. We will be examining contracts on specific types of assets, the characteristics of which give rise to cash flows to the holder of the asset.

Stock Indices and Dividends

We shall start here by assuming that our futures contract is a single stock futures, although the general principles are the same for stock index futures. For example, we could consider a portfolio that contains only one stock. In either case, assume that this stock pays a sure dividend of D_T on the expiration date. Now suppose that an investor buys the stock at a spot price of S_0 and sells a futures contract at a price of $f_0(T)$.

At expiration, the stock is sold at S_T, the dividend D_T is collected, and the futures contract generates a cash flow of $-(f_T(T) - f_0(T))$, which equals $-(S_T - f_0(T))$. Thus, the total cash flow at expiration is $D_T + f_0(T)$. This amount is known in advance; therefore, the current value of the portfolio must equal the present value of $D_T + f_0(T)$. The current portfolio value is simply the amount paid for the stock, S_0. Putting these results together gives

$$S_0 = (f_0(T) + D_T)(1 + r)^{-T},$$

or

$$f_0(T) = S_0(1 + r)^{T} - D_T.$$

Here we see that the futures price is the spot price compounded at the risk-free rate minus the dividend. Note that a sufficiently large dividend could bring the futures price down below the spot price.

To take our model one step closer to reality, let us assume that the stock pays several dividends. In fact, our stock could actually be a portfolio of stocks that is identical to an index such as the S&P 500. Suppose that N dividends will be paid during the life of the futures. Each dividend is denoted as D_j and is paid t_j years from today. Now suppose that we buy the stock and sell the futures. During the life of the futures, we collect each dividend and reinvest it in risk-free bonds earning the rate r. Thus, dividend D_1 will grow to a value of $D_1(1 + r)^{T-t_1}$ at expiration. By the expiration day, all dividends will have grown to a value of

$$\sum_{j=1}^{N} D_j(1+r)^{T-t_j},$$

which we shall write compactly as D_T. Thus, now we let D_T be the accumulated value at T of all dividends over the life of the futures plus the interest earned on them. In the previous example, we had only one dividend but D_T was still the same concept, the accumulated future value of the dividends. At expiration the stock is sold for S_T, and the futures is settled and generates a cash flow of $-(f_T(T) - f_0(T))$, which equals $-(S_T - f_0(T))$. Thus, the total cash flow at expiration is

$$S_T - (S_T - f_0(T)) + D_T,$$

or

$$f_0(T) + D_T.$$

This amount is also k.nown in advance, so its present value, discounted at the risk-free rate, must equal the current value of the portfolio, which is the value of the stock, S_0. Setting these terms equal and solving for $f_0(T)$ gives

$$f_0(T) = S_0(1 + r)^T - D_T.$$

Thus, the futures price is the spot price compounded at the risk-free rate minus the compound future value of the dividends. The entire process of buying the stock, selling a futures, and collecting and reinvesting dividends to produce a risk-free transaction is illustrated in Figure 9.1 for a stock that pays two dividends during the life of the futures. The total value accumulated at expiration is set equal to the total value today.

As an alternative to compounding the dividends, we can instead find the present value of the dividends and subtract this amount from the stock price before compounding it at the risk-free rate. In other words, the present value of the dividends over the life of the contract would be

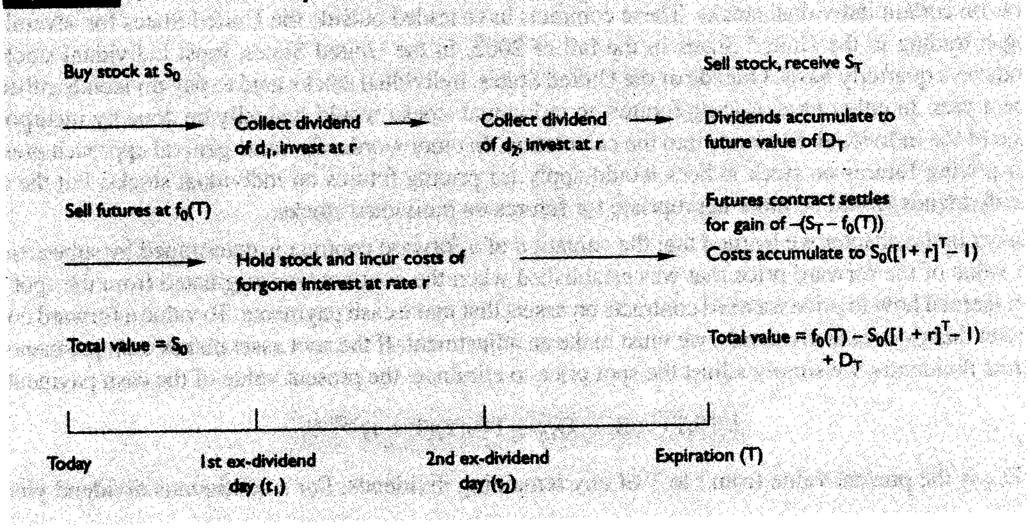
$$D_0 = \sum_{j=1}^{N} D_j(1+r)^{-t_j}.$$

The futures pricing formula would then be

$$f_0(T) = (S_0 - D_0)(1 + r)^T.$$

This approach is the one we took when pricing options. In fact, we encountered the concept of the present value of the dividends in Chapters 3, 4, and 5. We subtracted the present value of the dividends from the stock price and used the stock price in the options pricing model. We do the same here: subtract the present value of the dividends and insert this adjusted stock price into the futures pricing model.

FIGURE 9.1 The Cost of Carry Model with Stock Index Futures



A stock index is a weighted combination of securities, most of which pay dividends. In reality, the dividend flow is more or less continuous, although not of a constant amount. As we did with options, however, we can fairly safely assume a continuous flow of dividends at a constant yield, δ_c. Using r_c as the continuously compounded risk-free rate and S_0 as the spot price of the index, the model is written as

$$f_0(T) = S_0 e^{(r_c - \delta_c)T}.$$

It is worth noting that this formula is precisely what one would obtain if all of the assumptions of the Black-Scholes-Merton model were applied to futures.

This format makes an interpretation somewhat easier. Suppose that an investor is considering speculating on the stock market. There are two ways to do this: buy the stock index portfolio or buy the futures contract. If the portfolio is purchased, the investor receives dividends at a rate of δ_c. If the futures contract is purchased, the investor receives no dividends. The dividend yield enters the model as the factor $e^{-\delta cT}$, which is less than 1. Thus, the effect of dividends is to make the futures price lower than it would be without them. Note that the futures price will exceed (be less than) the spot price if the risk-free rate is higher (lower) than the dividend yield. Alternatively, the formula can be written as $f_0(T) = (S_0 e^{-\delta cT})e^{rcT}$, which can be interpreted as the dividend-adjusted stock price compounded at the risk-free rate.

Numerical Example Consider the following problem. A stock index is at 50, the continuously compounded risk-free rate is 8 percent, the continuously compounded dividend yield is 6 percent, and the time to expiration is 60 days, so T is 60/365 = 0.164. Then the futures price is

$$f_0(T) = 50e^{(0.08-0.06)(0.164)} = 50.16.$$

If the risk-free rate were 5 percent, the futures price would be

$$f_0(T) = 50e^{(0.05-0.06)(0.164)} = 49.92.$$

In Chapter 10 we shall see how arbitrage forces the market to adjust when the price does not equal the price given by the cost of carry model.

Most of the volume in futures contracts on equities is in stock index futures. There are, however, futures contracts on certain individual stocks. These contracts have traded outside the United States for several years and began trading in the United States in the fall of 2002. In the United States, most individual stocks pay dividends on a quarterly basis. Outside of the United States, individual stocks tend to pay dividends either once or twice a year. In either case, pricing futures on individual stocks would typically be done by incorporating the value of the individual dividends into the calculation. In other words, the same general approach presented here for pricing futures on stock indices would apply for pricing futures on individual stocks, but the use of discrete dividends would be more appropriate for futures on individual stocks.

Earlier in the chapter we learned that the valuation of a forward contract is determined by subtracting the present value of the forward price that was established when the contract was originated from the spot price. We then learned how to price forward contracts on assets that make cash payments. To value a forward contract on an asset that pays cash dividends, we must make an adjustment. If the spot asset makes cash payments such as discrete dividends, we simply adjust the spot price to eliminate the present value of the cash payments.

$$V_t(0,T) = S_t - D_{t,T} - F(0,T)(1 + r)^{-(T-t)},$$

where $D_{t,T}$ is the present value from t to T of any remaining dividends. For a continuous dividend yield, we have

$$V_t(0,T) = S_t e^{-\delta_c(T-t)} - F(0,T)e^{-r_c(T-t)},$$

where we observe that all of the discounting is done using the continuous form.

The basic idea behind pricing futures on a stock or stock index is generally applicable to pricing futures on bonds that pay coupons. But in practice, futures contracts based on bonds have certain features that complicate pricing beyond what we intend to cover in this chapter. These instruments are, however, covered in great detail in Chapter 10.

Foreign Currencies and Foreign Interest Rates: Interest Rate Parity

Interest rate parity is an important fundamental relationship between the spot and forward exchange rates and the interest rates in two countries. It is the foreign currency market's version of the carry arbitrage forward and futures pricing model. A helpful way to understand interest rate parity is to consider the position of someone who believes that a higher risk-free return can be earned by converting to a currency that pays a higher interest rate. For example, suppose that a French corporate treasurer wants to earn more than the euro interest rate and believes that he can convert euros to dollars and earn the higher U.S. rate. If the treasurer does so but fails to arrange a forward or futures contract to guarantee the rate at which the dollars will be converted back to euros, he runs the risk, not only of not earning the U.S. rate, but of earning less than the euro rate. If the dollar weakens while he is holding dollars, the conversion back to euros will be costly. This type of transaction is similar to going to a foreign country but not buying your ticket home until after you have been there a while. You are subject to whatever rates and conditions exist at the time the return ticket is purchased. Buying a round-trip ticket locks in the return price and conditions. Hence the corporate treasurer might wish to lock in the rate at which the dollars can be converted back to euros by selling a forward or futures contract on the dollar. But forward and futures prices will adjust so that the overall transaction will earn no more in euros than the euro interest rate. In effect, the cost of the return ticket will offset any interest rate gains while in the foreign currency. If it does not, there are arbitrage profits to be earned that will force prices to adjust appropriately. Consider the following situation involving euros and U.S. dollars, observed from the perspective of a European.

The spot exchange rate is S_0. This quote is in euros per U.S. dollar. The U.S. risk-free interest rate is r, and the holding period is T. You take $S_0(1 + \rho)^{-T}$ euros and buy $(1 + \rho)^{-T}$ dollars. Simultaneously, you sell one forward contract expiring at time T. The forward exchange rate is F_0, which is also in euros per dollar. You take your $(1 + \rho)^{-T}$ dollars and invest them in U.S. T-bills that have a return of r.

When the forward contract expires, you will have 1 dollar. This is because your $(1 + \rho)^{-T}$ dollars will have grown by the factor $(1 + \rho)^{T}$, so $(1 + \rho)^{-T}(1 + \rho)^{T} = 1$. Your forward contract obligates you to deliver the dollar, for which you receive F(0,T) euros. In effect, you have invested $S_0(1 + \rho)^{-T}$ and received F(0,T) euros. Since the transaction is riskless, your return should be the euro rate, r; that is,

$$F(0, T) = S_0(1 + \rho)^{-T}(1 + r)^{T}$$

This relationship is called *interest rate parity*.[1] It is sometimes expressed as

$$F(0, T) = S_0(1 + r)^{T} / (1 + \rho)^{T}.$$

Numerical Example Consider the following example from a European perspective. On June 9 of a particular year, the spot rate for dollars was 0.7908 euros. The U.S. interest rate was 5.84 percent, while the euro interest rate was 3.59 percent. The time to expiration was 90/365 = 0.2466. Recall, we have

$$F(0,T) = €0.7908(1.0584)^{-0.2466}(1.0359)^{0.2466} = 0.7866 \text{ euros.}$$

Thus, the forward rate should be about 0.7866 euros.

Interest rate parity can be confusing to some people because of the difference in the way the rates can be quoted. Many times you may see interest rate parity stated as $F(0,T) = S_0(1 + \rho)^{T}(1 + r)^{-T}$. This is correct if the spot rate is quoted in units of the foreign currency. In our euro example, we could have stated the spot rate as 1/€ 0.7908 = 1.2645 dollars per euro. In that case the forward rate would be stated in dollars per euro and the formula would be correctly given as $F(0,T) = S_0(1 + \rho)^{T}(1 + r)^{-T}$ with ρ being the foreign rate and r being the domestic rate. An easy way to remember this is that the factor for the interest rate for a given country multiplies by the spot quote stated in that country's currency. The other interest rate factor then appears with the −T in the exponent or simply in the denominator to the power T.

As we noted earlier in the chapter, when a forward rate is quoted in units of the domestic currency (for example, the dollar quoted per euros), when the forward rate is higher than the spot rate, the forward rate is said to be at a premium. Since the word "premium" tends to imply something higher, we have another reason to quote the currency in terms of the domestic currency. Had we quoted it the other way, a higher forward rate would imply a discount.

It has become common in discussions of international finance to interpret a forward premium (discount) as implying that the currency is expected to strengthen (weaken). Unfortunately, this is a mistaken belief. The principle of arbitrage is what gives rise to a forward premium or discount. If a person could convert his or her domestic currency to a foreign currency, lock in a higher risk-free return, and convert back with the currency risk-hedged, everyone would do this, which would erase any possibility of being able to earn a return better than the domestic risk-free rate. Any forward premium or discount is caused strictly by a difference in the interest rates of the two countries. If the domestic rate is lower and one forgoes the domestic risk-free return to earn the higher foreign risk-free return, the currency must sell at a forward rate proportional to the relative interest earned and given up. This has nothing to do with what people expect the spot rate to do in the future. People may have quite different beliefs about what might happen to spot rates, but we know they would agree that the forward rate must align with the spot rate by the proportional interest factors, or, in other words, by interest rate parity.

[1]There are also several common variations of this formula. It is sometimes approximated as $F(0,T) = S_0(1 + r - \rho)^{T}$ and sometimes as $F(0,T) = S_0(1 + rT)/(1 + \rho T)$ where T is days/360 or 365. The latter formula would be the case if r and ρ were in the form of LIBOR as discussed in previous chapters. Interest is added to the principal in the form of 1 + r(180/360). If the interest rates are continuously compounded, the formula would be $F(0,T) = S_0e^{(r_c - \rho_c)T}$

Although currency futures contracts are not traded as heavily as currency forwards, interest rate parity is also applicable to pricing those instruments, at least under the assumptions we have made here. Therefore,

$$f_0(T) = S_0(1 + r)^T / (1 + \rho)^T.$$

The value of currency forward contracts, following the same approach as used with stock index forwards, is

$$V_t(0,T) = S_t(1 + \rho)^{-(T-t)} - F(0,T)(1 + r)^{-(T-t)},$$

where ρ is the foreign interest rate.

Commodities and Storage Costs

In this section we consider commodity futures contracts and the impact of storage costs. For simplicity, assume that the storage cost for holding the underlying commodity, s, is paid at the end of the period. Now suppose that an investor buys the commodity at a spot price of S_0 and sells a futures contract at a price of $f_0(T)$.

At expiration, the commodity is sold at S_T; the storage cost, s, is paid; and the futures contract generates a cash flow of $-(f_T(T) - f_0(T))$, which equals $-(S_T - f_0(T))$. Thus, the total cash flow at expiration is $-s + f_0(T)$. This amount is known in advance; therefore, the current value of the portfolio must equal the present value of $-s + f_0(T)$. The current portfolio value is simply the amount paid for the commodity, S_0. Putting these results together gives

$$S_0 = (f_0(T) - s)(1 + r)^{-T},$$

or

$$f_0(T) = S_0(1 + r)^{-T} + s.$$

Here we see that the futures price is the spot price compounded at the risk-free rate plus the storage costs. We will explore commodity futures in more detail when risk premiums are discussed next.

PRICING MODELS AND RISK PREMIUMS

In this section we make the connection between forward or futures contract pricing and risk premiums. Recall from Chapter 1 (and other courses you may have taken) that a risk premium is the additional return expected in order to justify taking on the risk. You may already be familiar with asset pricing models, such as the famous Capital Asset Pricing Model, which give the relationship between expected return and risk. In this section, we want to determine whether futures and forward contracts provide risk premiums to parties that take positions in these contracts.

In the previous section we saw that forward and futures prices can differ but that the differences are usually quite small. To make things sound a little smoother, we shall stop referring to these contracts as both forward and futures contracts and shall simply refer to them as futures contracts. We shall assume that marking to market is done only on the expiration day, thus making a futures contract essentially a forward contract.

Before exploring risk premiums with futures contracts, let us review a few principles for determining spot prices from risk premiums and carry arbitrage.

Spot Prices, Risk Premiums, and Carry Arbitrage for Generic Assets

Let us first establish a simple framework for valuing generic spot assets. Let S_0 be the spot price, s be the cost of storing the asset over a period of time from 0 to T, and iS_0 be the interest forgone on S_0 dollars invested in the asset over that period of time.

First, let us assume that there is no uncertainty of the future asset price. Then, we can say that S_T will be the asset price at T for sure. Thus, the current price of the asset would have to be

$$S_0 = S_T - s - iS_0.$$

In other words, the asset price today would simply be the future price less the cost of storage and interest. No one would pay more than this amount, because storage and interest costs would wipe out any profit from holding it. No one would sell it for less than this amount, because someone would always be willing to pay more, up to the amount given in the above formula.

If we relax the assumption of a certain future asset price, then we must use the expected future asset price, $E(S_T)$. If investors are risk neutral, however, they will be willing to hold the asset without any expectation of receiving a reward for bearing risk. In that case,

$$S_0 = E(S_T) - s - iS_0.$$

In other words, the price today is the expected future price, less the storage and interest costs.

Most investors, however, are risk averse and consequently would not pay this much for the asset. They would pay a smaller amount, the difference being the risk premium. Let us denote that risk premium by $E(\phi)$. Then the current price would be

$$S_0 = E(S_T) - s - iS_0 - E(\phi).$$

In other words, in the real world of risk-averse investors, the current spot price is the expected future spot price, less any storage costs, less the interest forgone, and less the risk premium.

This statement is quite general and does not tell us anything about how the risk premium is determined. For financial assets, there are few if any storage costs, but in that case we just set $s = 0$. The above statement is a powerful reminder that asset prices must reflect expectations, the costs of ownership—both explicit and implicit—and a reward for bearing risk.

Let us make one final refinement. Recall that we discussed futures and forward contracts in which the underlying pays a cash return. Now let us make that assumption again, with the cash return being in the form of interest or a dividend. Let us capture this effect by reducing the interest opportunity cost by any such cash flows paid by the asset. So from now on let us remember that iS_0 is the interest forgone, less any cash flow—interest or dividends—received. We shall call this the net interest. Note that if the dividend or coupon interest rate is high enough, it can exceed the interest opportunity cost, making i be negative. This is not a problem and is, in fact, not all that rare. When a bond with a high coupon rate is held in an environment in which rates are low, the net interest can easily be negative.

The combination of storage costs and net interest, $s + iS_0$, is referred to as the cost of carry and is denoted with the Greek symbol θ (theta). The cost of carry is positive if the cost of storage exceeds the net interest and negative if the net interest is negative and large enough to offset the cost of storing. Sometimes, however, this concept is referred to as the carry. An asset that has a negative cost of carry, meaning that the net interest is a net inflow and exceeds the cost of storage, is said to have positive carry. An asset that has a positive cost of carry is said to have negative carry. For our purposes in this book, we shall refer to the concept as strictly the cost of carry.

For nonstorable goods, such as electricity, there would not necessarily be a relationship between today's spot price and the expected future spot price. Supply and demand conditions today and in the future would be independent. The risk of uncertain future supplies could not be reduced by storing some of the good currently owned. Large price fluctuations likely would occur. The cost of carry would be a meaningless concept.

At the other extreme, a commodity might be indefinitely storable. Stocks, metals, and some natural resources, such as oil, are either indefinitely or almost indefinitely storable. Their spot prices would be set in accordance with current supply and demand conditions, the cost of carry, investors' expected risk premia, and expected future supply and demand conditions.

For many agricultural commodities, limited storability is the rule. Grains have a fairly long storage life, while frozen concentrated orange juice has a more limited life. In the financial markets, Treasury bills, which mature in less than a year, have a short storage life. Treasury bonds, with their longer maturities, have a much longer storage life.

For any storable assets, the spot price is related to the expected future spot price by the cost of carry and the expected risk premium. We shall use this relationship to help understand forward and futures pricing.

Forward/Futures Pricing Revisited

Based on this chapter's previous discussions, we expand now our examination of forward and futures pricing. In particular, we explore some practical considerations such as the margin or other collateral requirements and the notion of convenience yield. Consider the following transaction: You buy the spot asset at a price of S_0, and sell a futures contract at a price of $f_0(T)$. At expiration, the spot price is S_T and the futures price is $f_T(T)$, which equals S_T. At expiration, you deliver the asset. The profit on the transaction is $f_0(T) - S_0$ minus the storage costs incurred and the opportunity cost of the funds tied up:

$$\Pi = f_0(T) - S_0 - s - iS_0 = f_0(T) - S_0 - \theta.$$

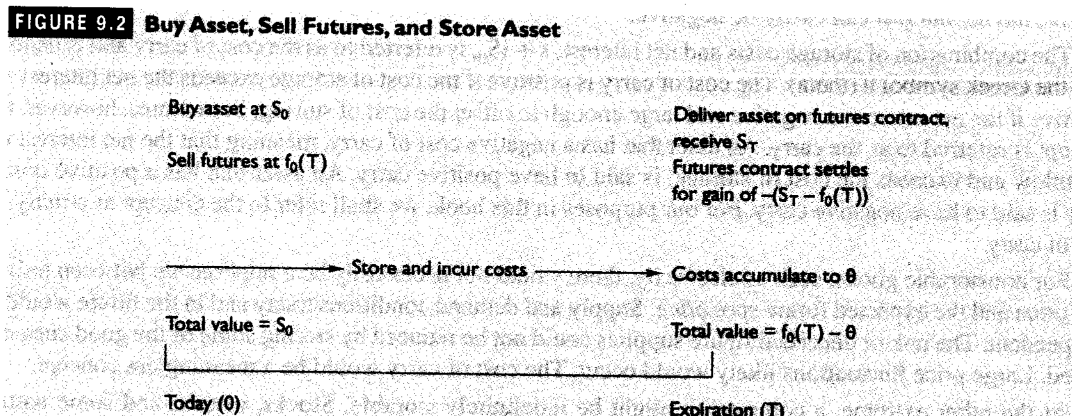
Since the expression $f_0(T) - S_0 - \theta$ involves no unknown terms, the profit is riskless, meaning the transaction should not generate a risk premium. The amount invested is S_0, the original price of the spot asset. The profit from the transaction is $f_0(T) - S_0 - \theta$, which should equal zero. Thus,

$$f_0(T) = S_0 + \theta.$$

The futures price equals the spot price plus the cost of carry. The cost of carry, therefore, is the difference between the futures price and the spot price and is related to the basis.[2] We shall say much more about the basis in Chapter 11.

An alternative interpretation of this transaction is seen in Figure 9.2. The value of the position when initiated is S_0; the value at expiration is $f_0(T) - \theta$. Since $f_0(T) - \theta$ is known when the transaction is initiated, the transaction is risk-free. So S_0 should equal the present value of $f_0(T) - \theta$ using the risk-free rate. But the present value adjustment has already been made since θ includes the interest lost on S_0 over the holding period. Thus, $f_0(T) = S_0 + \theta$.

FIGURE 9.2 Buy Asset, Sell Futures, and Store Asset



[2]The basis is usually defined as the spot price minus the futures price, and we shall use this definition in Chapter 11.

What makes this relationship hold? Assume that the futures price is higher than the spot price plus the cost of carry:

$$f_0(T) > S_0 + \theta.$$

Arbitrageurs will then buy the spot asset and sell the futures contract. This will generate a positive profit equal to $f_0(T) - S_0 - \theta$. Many arbitrageurs will execute the same transaction, which will put downward pressure on the futures price. When $f_0(T) = S_0 + \theta$, the opportunity to earn this profit will be gone.

Now suppose that the futures price is less than the spot price plus the cost of carry; that is,

$$f_0(T) < S_0 + \theta.$$

First, let us assume that the asset is a financial instrument. Then arbitrageurs will sell short the asset and buy the futures. When the instrument is sold short, the short seller will not incur the storage costs. Instead of incurring the opportunity cost of funds tied up in the asset, the short seller can earn interest on the funds received from the short sale. We shall examine arbitrage transactions in more detail in Chapter 10.

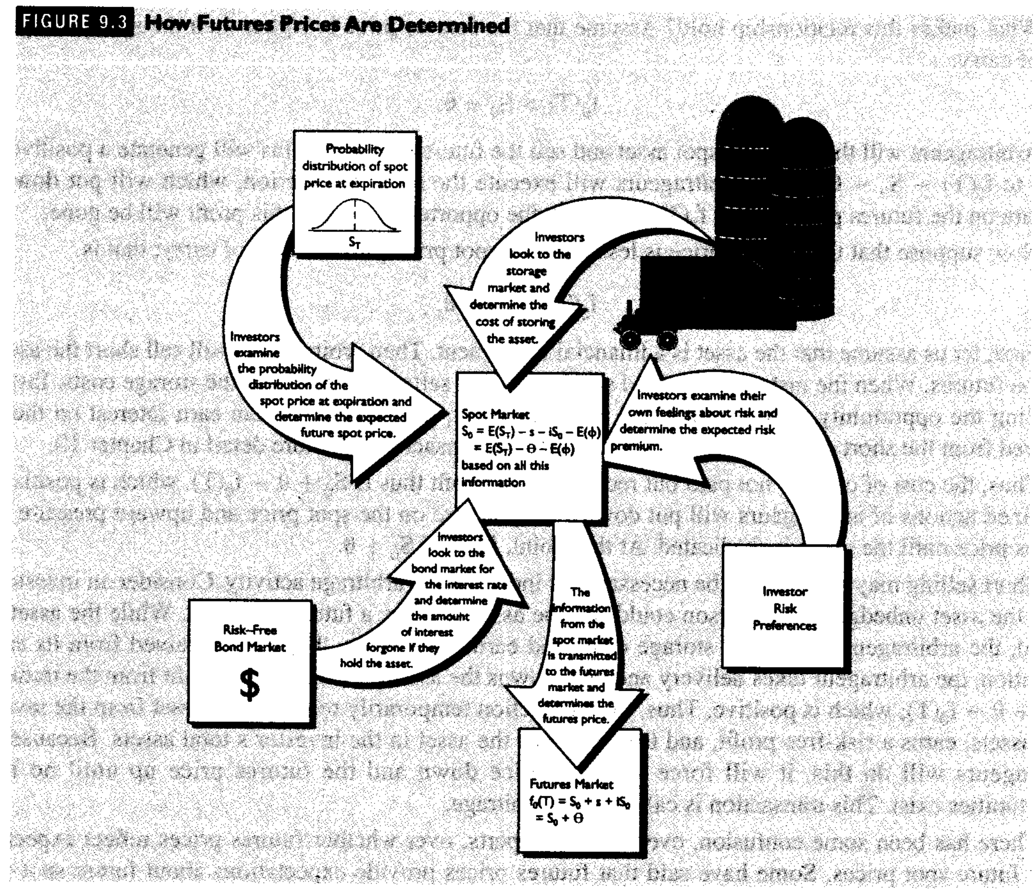
Thus, the cost of carry is not paid but received. The profit thus is $S_0 + \theta - f_0(T)$, which is positive. The combined actions of arbitrageurs will put downward pressure on the spot price and upward pressure on the futures price until the profit is eradicated. At that point, $f_0(T) = S_0 + \theta$.

Short selling may not actually be necessary for inducing the arbitrage activity. Consider an investor who holds the asset unhedged. That person could sell the asset and buy a futures contract. While the asset is not owned, the arbitrageur avoids the storage costs and earns interest on the funds received from its sale. At expiration, the arbitrageur takes delivery and again owns the asset unhedged. The profit from the transaction is $S_0 + \theta - f_0(T)$, which is positive. Thus, the transaction temporarily removes the asset from the investor's total assets, earns a risk-free profit, and then replaces the asset in the investor's total assets. Because many arbitrageurs will do this, it will force the spot price down and the futures price up until no further opportunities exist. This transaction is called quasi arbitrage.

There has been some confusion, even among experts, over whether futures prices reflect expectations about future spot prices. Some have said that futures prices provide expectations about future spot prices, while others have argued that futures prices reflect only the cost of carry. Still others have said that part of the time futures reveal expectations and part of the time they reveal the cost of carry. We shall more fully address the issue of whether futures prices reveal expectations in a later section; here we should note that both positions are correct. Because the futures price equals the spot price plus the cost of carry, the futures price definitely reflects the cost of carry. The spot price, however, reflects expectations. This is a fundamental tenet of spot pricing. Because the futures price will include the spot price, it too reflects expectations; however, it does so indirectly through the spot price. The overall process is illustrated in Figure 9.3.

So far we have assumed the small margin requirement imposed on futures traders is zero. Suppose now in the transaction involving the purchase of the asset at S_0 and sale of the futures at $f_0(T)$, the trader was required to deposit M dollars in a margin account. Let us assume that the M dollars will earn interest at the risk-free rate. Then at expiration the trader will have delivered the asset and received an effective price of $f_0(T)$ and have incurred the cost of carry of θ. In addition, the trader will be able to release the margin deposit of M dollars plus the interest on it. Since the total value at expiration, $f_0(T) - \theta + M +$ interest on M, is known in advance, the overall transaction remains risk-free. On the front end, however, the trader put up S_0 dollars to buy the asset and M dollars for the margin account. The present value of the total value at expiration should, therefore, equal $S_0 + M$. Obviously the present value of M plus the interest on M equals M. This means that S_0 must still equal $f_0(T) - \theta$, giving us our cost of carry model, $f_0(T) = S_0 + \theta$. If, however, interest is not paid on the margin deposit, the futures price can be affected by the loss of interest on the margin account. How the futures price is affected is not clear, because the trader who does the reverse arbitrage, selling or selling short the asset and buying the futures, also faces the same margin requirement. Does it really matter? Probably not. Large traders typically are able to deposit interest-bearing securities. The price we observe is almost surely being determined by large traders. So it seems reasonable to assume that the margin deposit is irrelevant to the pricing of futures.

FIGURE 9.3 How Futures Prices Are Determined



For storable assets, as well as for securities that do not pay interest or dividends, the cost of carry normally is positive. This would cause the futures price to lie above the current spot price. A market of this type is referred to as a contango.

Table 9.2 presents some spot and futures prices from a contango market. The example is for cotton traded on the New York Cotton Exchange. The cost of carry implied for the October contract is 41.60 − 36.75 = 4.85. Remember that this figure includes the interest forgone on the investment of 36.75 cents for a pound of cotton and the actual physical costs of storing the cotton from late September until the contract's expiration in October.

Table 9.2 An Example of a Contango Market

Cotton (New York Cotton Exchange)	
Expiration	Settlement Price (Cents per Pound)
Spot (September 26)	36.75
October	41.60
December	42.05
March	42.77
May	43.50
July	43.80
October	45.20
December	45.85

It would be convenient if fact always conformed to theory. If that were the case, we would never observe the spot price in excess of the futures price. In reality, spot prices sometimes exceed futures prices. A possible explanation is the convenience yield.

Convenience Yield We are seeking an explanation for the case in which the futures price is less than the spot price. If $f_0(T) = S_0 + \theta$ and $f_0(T) < S_0$, then $\theta < 0$. What type of market condition might produce a negative cost of carry?

Suppose that the commodity is in short supply; current consumption is unusually high relative to supplies of the good. This is producing an abnormally high spot price. The current tight market conditions discourage individuals from storing the commodity. If the situation is severe enough, the current spot price could be above the expected future spot price. If the spot price is sufficiently high, the futures price may lie below it. The relationship between the futures price and the spot price is then given as

$$f_0(T) = S_0 + \theta - \chi,$$

where χ (chi) is simply a positive value that accounts for the difference between $f_0(T)$ and $S_0 + \theta$. If χ is sufficiently large, the futures price will lie below the spot price. This need not be the case, however, since χ can be small.

The value χ often is referred to as the convenience yield. It is the premium earned by those who hold inventories of a commodity that is in short supply. By holding inventories of a good in short supply, one could earn an additional return, the convenience yield. Note that we are not saying that the commodity is stored for future sale or consumption. Indeed, when the spot price is sufficiently high, the return from storage is negative. There is no incentive to store the good. In fact, there is an incentive to borrow as much of the good as possible and sell short.

For some assets a convenience yield can be viewed as a type of nonpecuniary return. For example, consider a person who owns a house, which usually offers some potential for price appreciation but the expected gain is rarely sufficient to compensate for the risk involved. In some cases, the expected gain may be no more or even less than the risk-free rate. The house, however, provides a nonpecuniary yield, which is the utility from living in the house. The buyer of the house is normally willing to pay more, thereby reducing the expected return, for the right to live in the house.

When the commodity has a convenience yield, the futures price may be less than the spot price plus the cost of carry. In that case, the futures is said to be at less than full carry.

A market in which the futures price lies below the current spot price is referred to as backwardation or sometimes an inverted market. An example of a backwardation market is presented in Table 9.3.

Table 9.3 An Example of a Backwardation Market

	Soybeans (Chicago Board of Trade)
Expiration	**Price (Cents per Bushel)**
November	563.25
January	558.50
March	552.75
May	545.75
July	543.25
August	536.50
September	520.50
November	502.25

This example is taken from a day in the month of November. Since there is a November futures contract, the price of this contract is a good proxy for the spot price. Note that the November futures price of 563.25 is higher than all of the other futures prices. Clearly in this case, there is a convenience yield associated with the spot price. Soybeans are probably in short supply, but this shortage is likely to be alleviated over the next year because all of the futures prices are clearly below the spot price and the cost of carry.

It is not uncommon to see characteristics of both backwardation and contango in a market at the same time. Table 9.4 shows this case for soybean meal, from an example taken in the month of November. Note that the spot price is lower than the December contract price, which is lower than the January contract price. Note, however, that the May contract price is lower than the March contract price. This downward pattern continues, and the prices of all contracts expiring in September of the following year or later are lower than the spot price.

Table 9.4 An Example of a Simultaneous Backwardation and Contango Market

Soybean Meal (Chicago Board of Trade)

Expiration	Price (Cents per Bushel)
Spot (November 8)	159.50
December	163.50
January	164.40
March	164.80
May	163.00
July	162.40
August	161.10
September	158.30
October	154.90
December	155.10
January	154.10

Another factor that can produce backwardation in commodity markets is the inability to sell the commodity short and the reluctance on the part of holders of the commodity to sell it when its price is higher than it should be and replace it with an underpriced long futures contract. In the previous section we referred to this as quasi arbitrage. If quasi arbitrage is not executed in sufficient volume to bring the futures price to its theoretical fair price, then we could see backwardation. The spot price becomes too high and there is no one willing to sell the asset and replace it with a futures contract or no one able to sell short the asset.

Of course for financial assets, the cost of storage is negligible, and the supply of the commodity is fairly constant. Yet we still often observe an inverted market. For interest-sensitive assets like Eurodollars and Treasury bonds, either backwardation or contango can be observed. Later in this chapter we shall look at some other reasons why financial futures prices can be below spot prices.

With these concepts in mind, we now turn to an important and highly controversial issue in futures markets: Do futures prices contain a risk premium?

Futures Prices and Risk Premia

We have already discussed the concept of a risk premium in spot prices. No one would hold the spot commodity unless a risk premium was expected. Although investors do not always earn a risk premium, they must expect to do so on average. Is there a risk premium in futures prices? Are speculators in futures contracts rewarded, on average, with a risk premium? There are two schools of thought on the subject.

No Risk Premium Hypothesis Consider a simple futures market in which there are only speculators. The underlying commodity is the total amount of snowfall in inches in Vail, Colorado in a given week. The contracts are cash settled at expiration. Individuals can buy or sell contracts at whatever price they agree on. Similar derivatives exist on the over-the-counter market.

For example, suppose two individuals make a contract at a price of 30. If the total snowfall is above 30 at expiration, the trader holding the short position pays the holder of the long position a sum equal to the total snowfall minus 30. Though the ski resorts and merchants have exposures highly correlated with the level of snowfall, no one can actually "hold" the commodity, so there is no hedging or arbitrage.

Now suppose that after a period of several weeks, it is obvious that the longs are consistently beating the shorts. The shorts conclude that it is a good winter for skiing. Determined to improve their lot, those individuals who have been going short begin to go long. Of course, those who have been going long have no desire to go short. Now everyone wants to go long, and no one will go short. This drives up the futures price to a level at which someone finds it so high that it looks good to go short. Now suppose the price has been driven up so high that the opposite occurs: The shorts begin to consistently beat the longs. This causes the longs to turn around and go short. Ultimately an equilibrium must be reached in which neither the longs nor the shorts consistently beat the other side. In such a market, there is no risk premium. Neither side wins at the expense of the other.

In futures markets, this argument means that on average the futures price today equals the expected price of the futures contract at expiration; that is, $f_0(T) = E(f_T(T))$. Because the expected futures price at expiration equals the expected spot price at expiration, $E(f_T(T)) = E(S_T)$, we obtain the following result:

$$f_0(T) = E(S_T).$$

This is an extremely important and powerful statement. It says that *the futures price is the market's expectation of the future spot price.* If one wishes to obtain a forecast of the future spot price, one need only observe the futures price. In the language of economists, *futures prices are unbiased expectations of future spot prices.*

As an example, on September 26 of a particular year the spot price of silver was \$5.58 per troy ounce. The December futures price was \$5.64 per troy ounce. If futures prices contain no risk premium, the market is forecasting that the spot price of silver in December will be \$5.64. Futures traders who buy the contract at \$5.64 expect to sell it at \$5.64.

Figure 9.4 illustrates a situation that is reasonably consistent with this view. The May wheat futures contract is shown, along with the spot price for a period of 20 weeks prior to expiration. Both prices fluctuate, and the spot price exhibits a small risk premium, as suggested by the slight upward trend.[3] The futures price, however, follows no apparent trend.

We must caution, of course, that this is just an isolated case. The question of whether futures prices contain a risk premium, must be answered by empirical studies. First, however, let us turn to the arguments supporting the view that futures prices do contain a risk premium.

Asset Risk Premium Hypothesis If a risk premium were observed, we would see that

$$E(f_T(T)) > f_0(T).$$

The futures price would be expected to increase. Buyers of futures contracts at price $f_0(T)$ would expect to sell them at $E(f_T(T))$. Since futures and spot prices should converge at expiration, $E(f_T(T)) = E(S_T)$,

$$E(f_T(T)) = E(S_T) > f_0(T).$$

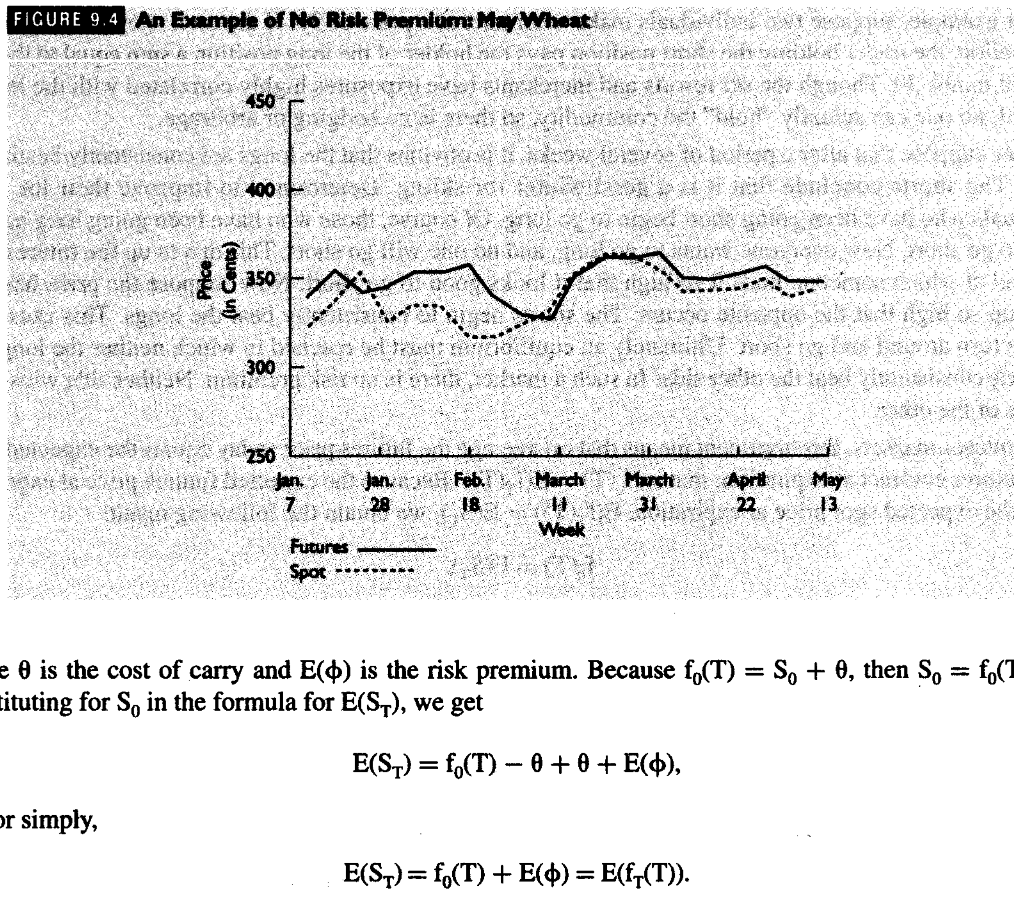
From this we conclude that the futures price is a low estimate of the expected future spot price.

Consider a contango market in which the cost of carry is positive. Holders of the commodity expect to earn a risk premium, $E(\phi)$, given by the following formula covered earlier in this chapter:

$$E(S_T) = E(S_0) + \theta + E(\phi),$$

[3]The upward drift, however, could also be due to the risk-free rate or possibly the storage costs.

FIGURE 9.4 An Example of No Risk Premium: May Wheat



where θ is the cost of carry and $E(\phi)$ is the risk premium. Because $f_0(T) = S_0 + \theta$, then $S_0 = f_0(T) - \theta$. Substituting for S_0 in the formula for $E(S_T)$, we get

$$E(S_T) = f_0(T) - \theta + \theta + E(\phi),$$

or simply,

$$E(S_T) = f_0(T) + E(\phi) = E(f_T(T)).$$

The expected futures price at expiration is higher than the current futures price by the amount of the risk premium. This means that buyers of futures contracts expect to earn a risk premium. They do not, however, earn a risk premium because the futures contract is risky. They earn the risk premium that existed in the spot market; it was merely transferred to the futures market.

Now, consider the silver example in the previous section. The spot price is \$5.58, and the December futures price is \$5.64. The interest lost on \$5.58 for two months is about \$0.05. Let us assume that the cost of storing silver for two months is \$0.01. Let us also suppose that buyers of silver expect to earn a \$0.02 risk premium. Thus, the variables are

$$S_0 = 5.58$$
$$f_0(T) = 5.64$$
$$\theta = 0.05 + 0.01 = 0.06$$
$$E(\phi) = 0.02.$$

The expected spot price of silver in December is

$$E(S_T) = S_0 + \theta + E(\phi) = 5.58 + 0.06 + 0.02 = 5.66.$$

Because the expected spot price of silver in December equals the expected futures price in December, $E(f_T(T)) = 5.66$. This can also be found as

$$E(f_T(T)) = f_0(T) + E(\phi) = 5.64 + 0.02 = 5.66.$$

Futures traders who buy the contract at 5.64 expect to sell it at 5.66 and earn a risk premium of 0.02. The futures price of 5.64 is an understatement of the expected spot price in December by the amount of the risk premium.

The process is illustrated as follows:

Spot:	Buy silver $5.58	Store and incur costs + $0.06	Expected risk premium + $0.02	Expected selling price = $5.66
Futures:	Buy silver futures $5.64		Expected risk premium + $0.02	Expected selling price = $5.66

The idea that futures prices contain a risk premium was proposed by two famous economists, Keynes (1930) and Hicks (1939). They argued that futures and spot markets are dominated by individuals who hold long positions in the underlying commodities. These individuals desire the protection afforded by selling futures contracts. That means they need traders who are willing to take long positions in futures. To induce speculators to take long positions in futures, the futures price must be below the expected price of the contract at expiration, which is the expected future spot price. Keynes and Hicks argued, therefore, that *futures prices are biased expectations of future spot prices,* with the bias attributable to the risk premium. This perspective is known as the risk premium hypothesis. Based on the carry arbitrage model, the risk premium in futures prices exists only because it is transferred from the spot market.

An example of such a case is shown in Figure 9.5, which illustrates a June S&P 500 futures contract. Both the spot and futures prices exhibit an upward trend.[4] Again, however, we must caution that this is only an isolated case.

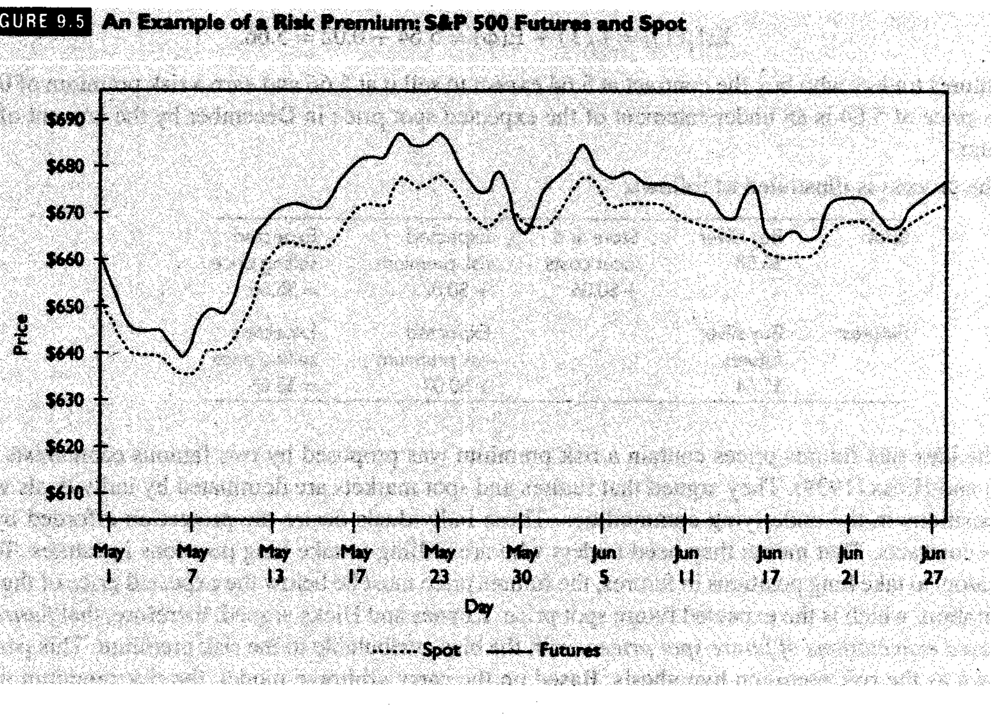
How can we explain the existence of a risk premium when we argued earlier that neither longs nor shorts would consistently win at the expense of the other side? The major difference in the two examples is the nature of the spot market. In the first example, in which the futures contract was on snowfall, there was no opportunity to take a "position" in the spot market. In fact, there was no spot market; futures traders were simply competing with one another in a pure gambling situation. When we allow for a spot market, we introduce individuals who hold speculative long positions in commodities. If the positions are unhedged, these individuals expect to earn a risk premium. If they are unwilling to accept the risk, they sell futures contracts. They are, in effect, purchasing insurance from the futures traders, and in so doing they transfer the risk and the risk premium to the futures markets.

This explanation is useful in seeing why futures markets should not be viewed as a form of legalized gambling. There are probably no greater risk takers in our society than farmers. They risk nearly all of their wealth on the output of their farms, which are subject to the uncertainties of weather, government interference, and foreign competition, not to mention the normal fluctuations of supply and demand. Farmers can lay off some or all of that risk by hedging in the futures markets. In so doing, they transfer the risk to parties willing to bear it. Yet no one would call farming legalized gambling. Nor would anyone criticize pension fund portfolio managers for gambling when they purchase stocks. Futures markets, and indeed all derivative markets, serve a purpose in facilitating risk transfer from parties not wanting it to parties willing to take it—for a price.

What about situations in which the hedgers buy futures? This would occur if hedgers were predominantly short the commodity. This would drive up futures prices, and futures prices would, on average, exhibit a downward trend as contracts approached expiration. Futures prices would overestimate future spot prices. Speculators who sold futures would earn a risk premium.

[4]Again the upward drift in the spot price could be due to the risk-free rate.

FIGURE 9.5 An Example of a Risk Premium: S&P 500 Futures and Spot



A market in which the futures price is below the expected future spot price is called normal backwardation, and one in which the futures price is above the expected future spot price is called normal contango. The choice of names for these markets is a bit confusing, and they must be distinguished from simply contango and backwardation.

$$\text{Contango: } S_0 < f_0(T)$$
$$\text{Backwardation: } f_0(T) < S_0$$
$$\text{Normal Contango: } E(S_T) < f_0(T)$$
$$\text{Normal Backwardation: } f_0(T) < E(S_T)$$

Because the spot price can lie below the futures price, which in turn can lie below the expected future spot price, we can have contango and normal backwardation simultaneously. We can also have backwardation and normal contango simultaneously.

Which view on the existence of a risk premium is correct? Since there is almost certainly a risk premium in spot prices, the existence of hedgers who hold spot positions means that the risk premium is transferred to futures traders. Thus, there would seem to be a risk premium in futures prices. If there are not enough spot positions being hedged, however, or if most hedging is being done by investors who are short in the spot market, there may be no observable risk premium in futures prices. Empirical studies have given us no clear-cut answer and suggest that the issue is still unresolved.

Put-Call-Forward/Futures Parity

Recall from Chapter 3 that we examined put-call parity: the relationship between put and call prices and the price of the underlying stock, the exercise price, the risk-free rate, and the time to expiration. We derived the equation by constructing a risk-free portfolio. Now we shall examine put-call-forward/futures parity with puts, calls, and forward or futures contracts. To keep things as simple as possible, we shall assume the risk-free rate is constant. This allows us to ignore marking to market and treat futures contracts as forward contracts. We assume the options are European.

The first step in constructing a risk-free portfolio is to recognize that if the exercise price is set to the futures price, a combination of a long call and a short put is equivalent to a (long) futures contract. In fact, a long-call/short-put combination is called a synthetic futures contract. A risk-free portfolio would consist of a long futures contract and a short synthetic futures contract. Selling the synthetic futures contract requires selling a call and buying a put.

Consider the portfolios illustrated in Table 9.5. We construct two portfolios, A and B. Examining the payoffs at expiration reveals that a long futures and a long put is equivalent to a long call and long risk-free bonds with a face value of the difference between the exercise price (X) and the futures price ($f_0(T)$). If $f_0(T)$ is greater than X, as it could easily be, then instead of long bonds, you will take out a loan for the present value of $f_0(T) - X$, promising to pay back $f_0(T) - X$ when the loan matures at the options' expiration. With equivalent payoffs, the value of portfolio A today must equal the value of portfolio B today. Thus,

$$P_e(S_0,T,X) = C_e(S_0,T,X) + (X - f_0(T))(1 + r)^{-T}.$$

Table 9.5 Put-Call-Forward/Futures Parity

Portfolio	Current Value	Payoffs from Portfolio Given Stock Price at Expiration: $S_T \le X$	$S_T > X$
A. Futures	0	$S_T - f_0(T)$	$S_T - f_0(T)$
Put	$P_e(S_0,T,X)$	$X - S_T$	0
		$X - f_0(T)$	$S_T - f_0(T)$
B. Call	$C_e(S_0,T,X)$	0	$S_T - X$
Bonds	$(X - f_0(T))(1 + r)^{-T}$	$X - f_0(T)$	$X - f_0(T)$
		$X - f_0(T)$	$S_T - f_0(T)$

We can, of course, write this several other ways, such as

$$C_e(S_0,T,X) - P_e(S_0,T,X) = (f_0(T) - X))(1 + r)^{-T}.$$

Notice that whether the put price exceeds the call price depends on whether the exercise price exceeds the futures price. If the parity is violated, it may be possible to earn an arbitrage profit. Of course, futures can be replaced with forwards under our assumptions.

Numerical Example A good instrument for examining put-call-forward/futures parity is the S&P 500 index options and futures. The options are European and trade on the CBOE, while the futures trade on the Chicago Mercantile Exchange. Suppose on May 14 the S&P 500 index was 1337.80 and the June futures was at 1339.30. The June 1340 call was at 40, and the put was at 39. The expiration date was June 18, and the risk-free rate was 4.56 percent.

Since there are 35 days between May 14 and June 18, the time to expiration is 35/365 = 0.0959. The left-hand side of the first put-call-futures parity equation is

$$P_e(S_0,T,X) = 39.$$

The right-hand side is

$$\begin{aligned} &C_e(S_0,T,X) + (X - f_0(T))(1 + r)^{-T} \\ &= 40 + (1340 - 1339.30)(1.0456)^{-0.0959} \\ &= 40.70. \end{aligned}$$

Thus, if you bought the put and futures, paying 39, and sold the call and the bond, receiving 40.70, the two portfolios will offset at expiration. So there is no risk and yet you earn a net gain of 40.70 − 39 = 1.70. Of course transaction costs might consume the difference.

PRICING OPTIONS ON FUTURES

In this section, we shall look at some principles of pricing options on futures. These principles are closely related to the principles of pricing ordinary options that were established in Chapters 3, 4, and 5 but also tie in with the principles of pricing futures contracts, as covered in this chapter. We will make the assumptions that the options and the futures contracts expire simultaneously and that the futures price equals the forward price.

Intrinsic Value of an American Option on Futures

The minimum value of an American call on a futures is its intrinsic value. We can formally state this as

$$C_a(f_0(T),T,X) \geq \text{Max}(0, f_0(T) - X),$$

where $\text{Max}(0, f_0(T) - X)$ is the intrinsic value. It is easy to see that this statement must hold for options on futures in the same way as for options on the spot. If the call price is less than the intrinsic value, the call can be bought and exercised. This establishes a long position in a futures contract at the price of X. The futures is immediately sold at the price of $f_0(T)$, and a risk-free profit is made.

Consider a March 1390 S&P 500 option on futures. The underlying futures price is 1401. The intrinsic value is Max(0, 1401 − 1390) = 11. The call is actually worth 49.20. The difference of 49.20 − 11 = 38.20 is the time value. Like the time value on an option on the spot, the time value here decreases as expiration approaches. At expiration, the call must sell for its intrinsic value.

The intrinsic value of an American put option on futures establishes its minimum value. This is stated as

$$P_a(f_0(T),T,X) \geq \text{Max}(0, X - f_0(T)),$$

where $\text{Max}(0, X - f_0(T))$ is the intrinsic value. Again, if this is not true, the arbitrageur can purchase the futures contract and the put, immediately exercise the put, and earn a risk-free profit.

The March 1405 S&P 500 put option on futures was priced at 44.60. The futures price was 1401. The minimum value is Max(0, 1405 − 1401) = 4.

The difference between the put price, 44.60, and the intrinsic value, 4, is the time value, 40.60. The time value, of course, erodes as expiration approaches. At expiration, the put is worth the intrinsic value.

Lower Bound of a European Option on Futures

The intrinsic values apply only to American options on futures. This is because early exercise is necessary to execute the arbitrage. As you should recall from our study of options on stocks, we can establish a lower bound for a European option.

Let us first look at the call option on futures. We construct two portfolios, A and B. Portfolio A consists of a single long position in a European call. Portfolio B consists of a long position in the futures contract and a long position in risk-free bonds with a face value of $f_0(T) - X$. Note that if X is greater than $f_0(T)$, this is actually a short position in bonds and thus constitutes a loan in which we pay back $X - f_0(T)$ at expiration. We do not really care whether we are borrowing or lending. As long as we keep the signs correct, we will obtain the desired result in either case. Table 9.6 presents the outcomes of these portfolios.

If $f_T(T) \leq X$, the call expires worthless. The futures contract is worth $f_T(T) - f_0(T)$, and the bonds are worth $f_0(T) - X$; thus, portfolio B is worth $f_T(T) - X$. If $f_T(T) > X$, the call is worth $f_T(T) - X$, the intrinsic

Table 9.6 The Lower Bound of a European Call Option on Futures: Payoffs at Expiration of Portfolios A and B

			Payoffs from Portfolio Given Futures Price at Expiration	
Portfolio	Instrument	Current Value	$f_T(T) \le X$	$f_T(T) > X$
A	Long call	$C_e(f_0(T), T, X)$	0	$f_T(T) - X$
B	Long futures	0	$f_T(T) - f_0(T)$	$f_T(T) - f_0(T)$
	Bond	$(f_0(T) - X)(1 + r)^{-T}$	$+f_0(T) - X$	$+f_0(T) - X$
			$f_T(T) - X$	$f_T(T) - X$

value, and portfolio B is still worth $f_T(T) - X$. As you can see, portfolio A does at least as well as portfolio B in all cases; therefore, its current value should be at least as high as portfolio B's. We can state this as

$$C_e(f_0(T),T,X) \ge (f_0(T) - X)(1 + r)^{-T}.$$

Because an option cannot have negative value,

$$C_e(f_0(T),T,X) \ge \text{Max}[0,(f_0(T) - X)(1 + r)^{-T}].$$

Note that we used an important result from earlier in this chapter: The value of a futures contract when initially established is zero. Thus, portfolio B's value is simply the value of the risk-free bonds.

This result establishes the lower bound for a European call on the futures. Remember that a European call on the spot has a lower bound of

$$C_e(S_0,T,X) \ge \text{Max}[0,S_0 - X(1 + r)^{-T}].$$

As we saw earlier in this chapter, in the absence of dividends on the spot instrument, the futures price is

$$f_0(T) = S_0(1 + r)^{-T}.$$

Making this substitution for $f_0(T)$, we see that these two lower bounds are equivalent. In fact, if the option and futures expire simultaneously, a European call on a futures is equivalent to a European call on the spot. This is because a European call can be exercised only at expiration, at which time the futures and spot prices are equivalent.

As an example of the lower bound, let us look at the March 1390 S&P 500 call option on futures on January 31 of a leap year. The option expires on March 16; thus, there are 45 days remaining and $T = 45/365 = 0.1233$. The futures price is 1401. The risk-free rate is 5.58 percent. The lower bound is

$$C_e(f_0,T,X) \ge \text{Max}[0,(1401 - 1390)(1.0558)^{-0.1233}] = 10.93.$$

The actual call price is 49.20.

Note, however, that the lower bound established here is slightly less than the intrinsic value of 11. This should seem unusual. For ordinary equity options, the lower bound of $\text{Max}[0,S_0 - X(1 + r)^{-T}]$ exceeds the intrinsic value of $\text{Max}(0, S_0 - X)$. For options on futures, however, this is not necessarily so. As we shall see in a later section, this explains why some American call (and put) options on futures are exercised early.

Now let us look at the lower bound for a European put option on a futures. Again we shall establish two portfolios, A and B. Portfolio A consists of a long position in the put. Portfolio B consists of a short position in the futures contract and a long position in risk-free bonds with a face value of $X - f_0(T)$. Again, if $f_0(T)$ is greater than X, this is actually a short position in bonds, or taking out a loan. Table 9.7 illustrates the current value and future payoff of each portfolio.

Table 9.7 The Lower Bound of a European Put Option on Futures: Payoffs at Expiration of Portfolios A and B

			Payoffs from Portfolio Given Futures Price at Expiration	
Portfolio	Instrument	Current Value	$f_T(T) < X$	$f_T(T) \geq X$
A	Long put	$P_e(f_0(T), T, X)$	$X - f_T(T)$	0
B	Short futures	0	$-(f_T(T) - f_0(T))$	$-(f_T(T) - f_0(T))$
	Bond	$+ (X - f_0(T))(1 + r)^{-T}$	$+ X - f_0(T)$	$+ X - f_0(T)$
			$X - f_T(T)$	$X - f_T(T)$

By now you should be able to explain each outcome. If $f_T(T) < X$, the put is exercised, so portfolio A is worth $X - f_T(T)$. If $f_T(T) \geq X$, the put expires worthless. In both cases, the futures contract in portfolio B is worth $-(f_T(T) - f_0(T))$ and the bonds are worth $X - f_0(T)$, for a total of $X - f_T(T)$. Portfolio A does at least as well as portfolio B. Thus, the current value of A should be at least as great as the current value of B,

$$P_e(f_0(T),T,X) \geq (X - f_0(T))(1 + r)^{-T}.$$

Because the option cannot have a negative value,

$$P_e(f_0(T),T,X) \geq \text{Max}[0, (X - f_0(T))(1 + r)^{-T}].$$

As we saw with calls, we can substitute S_0 for $f_0(T)(1 - r)^{-T}$ and see that the lower bound for a put option on a futures is the same as that for a put option on the spot. As is true for calls, European put options on futures in which the put and the futures expire simultaneously are equivalent to options on the spot.

As an example, let us look at the March 1405 S&P 500 put option on futures on January 31, 2000. The futures price is 1401, the time to expiration is 0.1233, and the risk-free rate is 5.58 percent. The lower bound is

$$P_e(f_0(T),T,X) \geq \text{Max}[0,(1405 - 1401)(1.0558)^{-0.1233}] = 3.97.$$

The actual price of the put is 44.60. As we saw for equity puts, the European lower bound will be less than the American intrinsic value. Thus, the actual minimum price of this American put is its intrinsic value of $1405 - 1401 = 4$.

Put-Call Parity of Options on Futures

We have looked at put-call parity for options on stocks. We can also establish a put-call parity rule for options on futures.

First let us construct two portfolios, A and B. Portfolio A will consist of a long futures and a long put on the futures. This can be thought of as a protective put. Portfolio B will consist of a long call and a long bond with a face value of the exercise price minus the futures price. If X is greater than $f_0(T)$, this is indeed a long position in a bond. If $f_0(T)$ is greater than X, then we are simply issuing bonds with a face value of $f_0(T) - X$. In either case, the cash flow of the bond will be $X - f_0(T)$ when it matures on the option expiration day. The payoffs are illustrated in Table 9.8.

As we can see, the two portfolios produce the same result. If the futures price ends up less than the exercise price, both portfolios end up worth $X - f_0(T)$. If the futures price ends up greater than the exercise price, both portfolios end up worth $f_T(T) - f_0(T)$. Thus, portfolio B is also like a protective put and its current

Table 9.8 Put-Call Parity of Options on Futures

Portfolio	Instrument	Current Value	Payoffs from Portfolio Given Futures Price at Expiration: $f_T(T) \le X$	$f_T(T) > X$
A	Long Futures	0	$f_T(T) - f_0(T)$	$f_T(T) - f_0(T)$
	Long Put	$P_e(f_0(T), T, X)$	$X - f_T(T)$	0
		0	$X - f_0(T)$	$f_T(T) - f_0(T)$
B	Long Call	$C_e(f_0(T), T, X)$	0	$f_T(T) - X$
	Bonds	$(X - f_0(T))(1 + r)^{-T}$	$X - f_0(T)$	$X - f_0(T)$
			$X - f_0(T)$	$f_T(T) - f_0(T)$

value must equal the current value of portfolio A. Since the value of the long futures in portfolio A is zero, we conclude that

$$P_e(f_0(T),T,X) = C_e(f_0(T),T,X) + (X - f_0(T))(1 + r)^{-T}.$$

As with put-call parity for options on spot assets, we can write this several different ways, isolating the various terms. Note the similarity between put-call parity for options on futures and put-call parity for options on the spot:

$$P_e(f_0(T),T,X) = C_e(f_0(T),T,X) - S_0 + X(1 + r)^{-T}.$$

Because the futures price must equal $S_0(1 + r)^T$, these two versions of put-call parity are equivalent. As we stated earlier, in many ways the options themselves are equivalent.

Let us look at the March 1400 puts and calls on the S&P 500 futures on January 31. As we saw in Chapter 3, we can calculate the put price and compare it to the actual market price or calculate the call price and compare it to the actual market price. Here, we shall calculate the put price. The call price is $43.40. The other input values given earlier are $f_0 = 1401$, $r = 0.0558$, and $T = 0.1233$. The put price should be

$$P_e(f_0(T),T,X) = 43.40 + (1400 - 1401)(1.0558)^{-0.1233} = 42.41.$$

The actual put price was 42.40. This is very close, but we should expect a difference because these are American options and the formula is for European options. The formula price should be less than the market price, but in this case it is not. The effect of transaction costs, however, might explain the difference.

Early Exercise of Call and Put Options on Futures

Recall that in the absence of dividends on a stock, a call option on the stock would not be exercised early; however, a put option might be. With an option on a futures contract, either a call or a put might be exercised early. Let us look at the call.

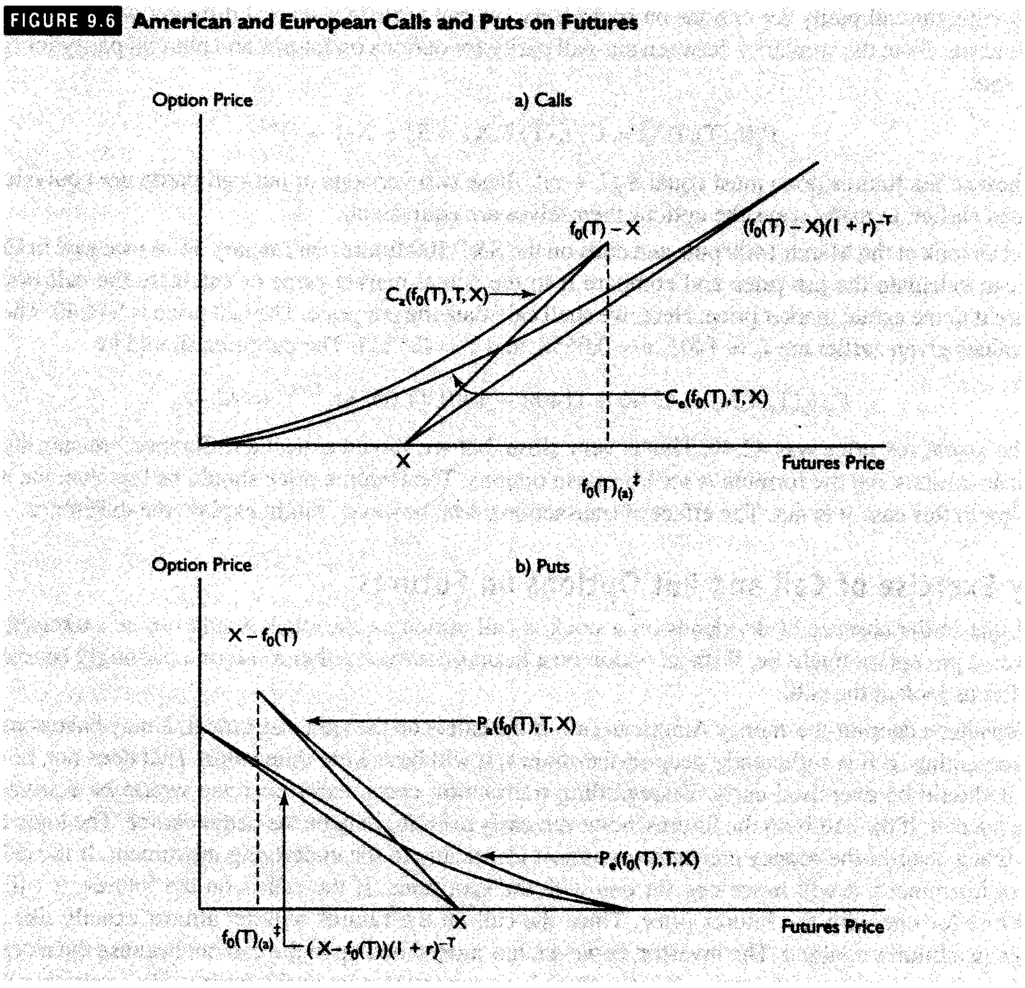
Consider a deep-in-the-money American call. If the call is on the spot instrument, it may have some time value remaining. If it is sufficiently deep-in-the-money, it will have little time value. That does not, however, mean it should be exercised early. Disregarding transaction costs, early exercise would be equivalent to selling the call. If the call is on the futures, however, early exercise may be the better choice. The logic behind this is that a deep-in-the-money call behaves almost identically to the underlying instrument. If the call is on the spot instrument, it will move one for one with the spot price. If the call is on the futures, it will move nearly one for one with the futures price. Thus, the call on the futures will act almost exactly like a long position in a futures contract. The investor, however, has money tied up in the call but because the margin can be met by depositing interest-earning T-bills, there is no money tied up in the futures. By exercising the call and replacing it with a long position in the futures, the investor obtains the same opportunity to profit but frees up the funds tied up in the call. If the call were on the spot instrument, we could not make the same argument. The call may behave in virtually the same manner as the spot instrument, but the latter also requires the commitment of funds.

From an algebraic standpoint, the early-exercise problem is seen by noting that the minimum value of an in-the-money European call, $(f_0(T) - X)(1 + r)^{-T}$, is less than the value of the call if it could be exercised, $f_0(T) - X$. The European call cannot be exercised, but if it were an American call, it could be.

These points are illustrated in panel a of Figure 9.6. The European call option on futures approaches its lower bound of $(f_0(T) - X)(1 + r)^{-T}$. The American call option on futures approaches its minimum value, its intrinsic value of $f_0(T) - X$, which is greater than the European lower bound. There is a futures price, $f_0(T)_{(a)}^{\ddagger}$, at which the American call will equal its intrinsic value. Above that price, the American call will be exercised early. Recall that for calls on spot instruments, the European lower bound was higher than the intrinsic value. Thus, there was no early exercise premium—provided, of course, that the underlying asset paid no dividends.

For put options on futures, the intuitive and algebraic arguments work similarly. Deep-in-the-money American puts on futures tend to be exercised early. Panel b of Figure 9.6 illustrates the case for puts. The price of European put options on futures approaches its lower bound of $(X - f_0(T))(1 + r)^{-T}$, while the price

FIGURE 9.6 American and European Calls and Puts on Futures



of the American put option on futures approaches its intrinsic value of X − $f_0(T)$. The American intrinsic value is greater than the European lower bound. There is a price, $f_0(T)_{(a)}^{\ddagger}$, at which the American put option on futures would equal its intrinsic value. Below this price, the American put would be exercised early.

Although these instruments are options on futures, it is conceivable that one would be interested in an option on a forward contract. Interestingly, these options would not be exercised early. This is because when exercised, a forward contract is established at the price of X. A forward contract is not marked to market, however, so the holder of a call does not receive an immediate cash flow of F(0,T) − X. Instead, that person receives only a position with a value, as we learned earlier in this chapter, of $(F(0,T) - X)(1 + r)^{-T}$. This amount, however, is the lower bound of a European call on a forward contract. Early exercise cannot capture a gain over the European call value; thus, it offers no advantage. Similar arguments hold for early exercise of a European put on a forward contract.

Black Futures Option Pricing Model

Fischer Black (1976) developed a variation of his own Black-Scholes-Merton model for pricing European options on futures. Using the assumption that the option and the futures expire simultaneously, the Black model gives the option price as follows:

$$C = e^{-r_cT}[f_0(T)N(d_1) - XN(d_2)],$$

where

$$d_1 = \frac{\ln(f_0(T)/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Here σ is the volatility of the futures. Note that the expression for d_1 does not contain the continuously compounded risk-free rate, r_c, as it does in the Black-Scholes-Merton model. That is because the risk-free rate captures the opportunity cost of funds tied up in the underlying asset. If the option is on a futures contract, no funds are invested in the futures, and therefore there is no opportunity cost. The price of the call on the futures, however, will be the same as the price of the call on the spot. This is because when the call on the futures expires, it is exercisable into a futures position, which is immediately expiring. Thus, the call on the futures, when exercised, establishes a long position in the spot asset.

To prove that the Black futures option pricing model gives the same price as the Black-Scholes-Merton model for an option on the spot, notice that in the Black model $N(d_1)$ is multiplied by $f_0(T)e^{-r_cT}$, while in the Black-Scholes-Merton model it is multiplied by S_0. We learned earlier in this chapter that with no dividends on the underlying asset, the futures price will equal $S_0e^{r_cT}$, Thus, if we substitute $S_0^{r_cT}$ for $f_0(T)$ in the Black model, the formula will be the same as the Black-Scholes-Merton formula.[5]

[5]This may not be obvious in the formula for d_1. If we substitute $S_0e^{r_cT}$ for $f_0(T)$ in the above formula for d_1, we obtain

$$d_1 = \frac{\ln(S_0e^{r_cT}/X) + (\sigma^2/2)T}{\sigma\sqrt{T}}.$$

The expression $\ln(S_0e^{r_cT}/X)$ is equivalent to $\ln S_0 + \ln e^{r_cT} - \ln X$. Now $\ln e^{r_cT} = r_cT$, so d_1 becomes

$$d_1 = \frac{\ln(S_0/X) + (r_c + \sigma^2/2)T}{\sigma\sqrt{T}},$$

which is d_1 from the Black-Scholes-Merton formula.

We know that in the presence of dividends, the futures price is given by the formula $f_0(T) = S_0e^{(r_c - \delta_c)T}$. Thus, $S_0 = f_0(T)e^{-(r_c - \delta_c)T}$. If we substitute this expression for S_0 into the Black-Scholes-Merton model, we obtain the Black futures option pricing model if the underlying spot asset—in this case, a stock—pays dividends. The dividends do not show up in the Black model, however, so we need not distinguish the Black model with and without dividends. Dividends do affect the call price, but only indirectly, as the futures price captures all the effects of the dividends.

Another useful comparison of the Black and Black-Scholes-Merton models is to consider how the Black-Scholes-Merton model might be used as a substitute for the Black model. Suppose that we have available only a computer program for the Black-Scholes-Merton model, but we want to price an option on a futures contract. We can do this easily by using the version of the Black-Scholes-Merton model in Chapter 5 that had a continuous dividend yield, and inserting the risk-free rate for the dividend yield and the futures price for the spot price. The risk-free rate minus the dividend yield is the cost of carry, so it will equal zero. The Black-Scholes-Merton formula will then be pricing an option on an instrument that has a price of $f_0(T)$ and a cost of carry of zero. This is precisely what the Black model prices: an option on an instrument—in this case, a futures contract—with a price of $f_0(T)$ and a cost of carry of zero. Remember that the futures price reflects the cost of carry on the underlying spot asset, but the futures itself does not have a cost of carry because there are no funds tied up and no storage costs.

Let us now use the Black model to price the March 1400 call option on the S&P 500 futures. Recall that the futures price is 1401, the exercise price is 1400, the time to expiration is 0.1233, and the risk-free rate is $\ln(1.0558) = 0.0543$. We now need only the standard deviation of the continuously compounded percentage change in the futures price. For illustrative purposes, we shall use 21 percent as the standard deviation. Table 9.9 presents the calculations.

Table 9.9 Calculating the Black Option on Futures Price

$f_0(T) = 1401$ $\quad X = 1400 \quad$ $r_c = 0.0543$ $\quad \sigma = 0.21 \quad$ $T = 0.1233$

1. Compute d_1 $\quad d_1 = \dfrac{\ln(1401/1400) + ((0.21)^2/2)0.1233}{0.21\sqrt{0.1233}} = 0.0466.$
2. Compute d_2 $\quad d_2 = 0.0466 - 0.21\ \sqrt{0.1233}\ = 0.0271.$
3. Look up $N(d_1)$ $\quad N(0.05) = 0.5199.$
4. Look up $N(d_2)$ $\quad N(-0.03) = 0.4880.$
5. Plug into formula for C $\quad C = e^{-(0.0543)(0.1233)}[1401(0.5199) - 1400(0.4880)] = 44.88.$

The actual value of the call is \$43.40. Thus, the call would appear to be underpriced. As we showed in Chapter 5, an arbitrageur could create a risk-free portfolio by selling the underlying instrument and buying the call. The hedge ratio would be $e^{-r_cT}N(d_1)$ futures contracts for each call; however, remember that the model gives the European option price, so we expect it to be less than the actual American option price.

Your software, BSMbin7e.xls and BSMbwin7e.exe, introduced in Chapter 5, can be used to obtain prices of options on futures for the Black model. Simply insert the value you used for the risk-free rate for the continuously compounded dividend yield and insert the futures price for the underlying asset price.

In Chapter 5 when we studied the Black-Scholes-Merton model, we carefully examined how the model changes when any of the five underlying variables changes. Many of these effects were referred to with Greek names like delta, gamma, theta, vega, and rho. Since the Black model produces the same price as the Black-Scholes-Merton model, we will get the same effects here. The only difference is that with the Black model we express these results in terms of the futures price, rather than the spot or stock price. For any of the formulas in which S_0 appears, such as the gamma, vega, and theta, we simply replace S_0 with $f_0(T)^{-r_cT}$. In the case of the delta, we must redefine delta as the change in the call price for a change in the futures price. For an option on a stock, we saw that the delta is $N(d_1)$. For an option on a futures, the delta is $e^{-r_cT}N(d_1)$.

We can easily develop a pricing model for European put options on futures from the Black model and put-call parity. Using the continuously compounded version, put-call parity is expressed as $C - P = (f_0(T) - X)e^{-r_cT}$. Rearranging this expression to isolate the put price gives

$$P = C - (f_0(T) - X)e^{-r_cT}.$$

Now, we can substitute the Black European call option on futures pricing model for C in put-call parity and rearrange the terms to obtain the Black European put option on futures pricing model,

$$P = Xe^{-r_cT}[1 - N(d_2)] - f_0(T)e^{-r_cT}[1 - N(d_1)].$$

Some end-of-chapter problems will allow us to use this model and examine it further.

Earlier we noted that even in the absence of dividends, American calls on futures might be exercised early. Like options on the spot, American puts on futures might be exercised early. The Black model does not price American options, and we cannot appeal to the absence of dividends, as we could for some stocks, to allow us to use the European model to price an American option. Unfortunately, American option on futures pricing models are fairly complex and beyond the scope of this book. It is possible, however, to price American options on futures using the binomial model. We would fit the binomial tree to the spot price, derive the corresponding futures price for each spot price, and then price the option using the futures prices in the tree.

In addition to the problem of using a European option pricing model to price American options, the Black model has difficulty pricing the most actively traded options on futures, Treasury bond options on futures. That problem is related to the interest rate component. The Black model, like the Black-Scholes-Merton model, makes the assumption of a constant interest rate. This generally is considered an acceptable assumption for pricing options on commodities and sometimes even stock indices. It is far less palatable for pricing options on bonds. There is a fundamental inconsistency in assuming a constant interest rate while attempting to price an option on a futures that is on an underlying Treasury bond, whose price changes because of changing interest rates. More appropriate models are available but this is an advanced topic that we do not address in this book.

QUESTIONS AND PROBLEMS

1. Assume that there is a forward market for a commodity. The forward price of the commodity is \$45. The contract expires in one year. The risk-free rate is 10 percent. Now, six months later, the spot price is \$52. What is the forward contract worth at this time? Explain why this is the correct value of the forward contract in six months even though the contract does not have a liquid market like a futures contract.

2. On a particular day the S&P 500 futures settlement price was 899.30. You buy one contract at around the close of the market at the settlement price. The next day, the contract opens at 899.70 and the settlement price at the close of the day is 899.10. Determine the value of the futures contract at the opening, an instant before the close, and after the close. Remember that the S&P futures contract has a $500 multiplier.
3. On July 10 a farmer observes that the spot price of corn is $2.735 per bushel and the September futures price is $2.76. The farmer would like a prediction of the spot price in September but believes the market is dominated by hedgers holding long positions in corn. Explain how the farmer would use this information in a forecast of the future price of corn.
4. Construct an arbitrage example involving an asset that can be sold short, and use it to explain the cost of carry model for pricing futures.
5. Why is the value of a futures or forward contract at the time it is purchased equal to zero? Contrast this with the value of the corresponding spot commodity.
6. Use the following data from January 31 of a particular year for a group of March 480 options on futures contracts to answer parts *a* through *g*.

Futures price: 483.10
Expiration: March 18
Risk-free rate: 0.0284 percent (simple)
Call price: 6.95
Put price: 5.25

   a. Determine the intrinsic value of the call.
   b. Determine the time value of the call.
   c. Determine the lower bound of the call.
   d. Determine the intrinsic value of the put.
   e. Determine the time value of the put.
   f. Determine the lower bound of the put.
   g. Determine whether put-call parity holds.
7. On September 12, a stock index futures contract was at 423.70. The December 400 call was at 26.25, and the put was at 3.25. The index was at 420.55. The futures and options expire on December 21. The discrete risk-free rate was 2.75 percent. Determine if the futures and options are priced correctly in relation to each other. If they are not, construct a risk-free portfolio and show how it will earn a rate better than the risk-free rate.
8. On a particular day, the September S&P 500 stock index futures was priced at 960.50. The S&P 500 index was at 956.49. The contract expires 73 days later.
   a. Assuming continuous compounding, suppose the risk-free rate is 5.96 percent, and the dividend yield on the index is 2.75 percent. Is the futures overpriced or underpriced?
   b. Assuming annual compounding, suppose the risk-free rate is 5.96 percent, and the future value of dividends on the index is $5.27. Is the futures overpriced or underpriced?
9. The following information was available:
   Spot rate for Japanese yen: $0.009313
   730-day forward rate for Japanese yen: $0.010475 (assume a 365-day year)
   U.S. risk-free rate: 7.0 percent
   Japanese risk-free rate: 1.0 percent

a. Assuming annual compounding, determine whether interest rate parity holds and, if not, suggest a strategy.
b. Assuming continuous compounding, determine whether interest rate parity holds and, if not, suggest a strategy.

10. Consider the wheat example in problem 24. The interest forgone on money tied up in a bushel until expiration is 0.03, and the cost of storing the wheat is 0.0875 per bushel. The risk premium is 0.035 per bushel.
    a. What is the expected price of wheat on the spot market in December?
    b. Show how the futures price is related to the spot price.
    c. Show how the expected spot price at expiration, your answer in part a, is related to the futures price today.
    d. Show how the expected futures price at expiration is related to the futures price today.
    e. Explain who earns the risk premium and why.
11. Suppose there is a commodity in which the expected future spot price is $60. To induce investors to buy futures contracts, a risk premium of $4 is required. To store the commodity for the life of the futures contract would cost $5.50. Find the futures price.
12. (Concept Problem) Suppose that a futures margin account pays interest but at a rate that is less than the risk-free rate. Consider a trader who buys the asset and sells a futures to form a risk-free hedge. Would the trader believe the futures price should be lower or higher than if the margin account paid interest at the risk-free rate?
13. Assume a standard deviation of 8 percent, and use the Black model to determine if the call option in problem 6 is correctly priced. If not, suggest a riskless hedge strategy.
14. Using the information in problem 13, calculate the price of the put described in problem 6, using the Black model for pricing puts.
15. Suppose you observe a one-year futures price of $100, the futures option strike price of $90, and a 5 percent interest rate (annual compounding). If the futures option call price is quoted at $9.40, identify any arbitrage and explain how it would be captured.
16. The put-call parity rule can be expressed as $C - P = (f_0(T) - X)(1 + r)^{-T}$. Consider the following data: $f_0(T) = 102$, $X = 100$, $r = 0.1$, $T = 0.25$, $C = 4$, and $P = 1.75$. A few calculations will show that the prices do not conform to the rule. Suggest an arbitrage strategy and show how it can be used to capture a risk-free profit. Assume that there are no transaction costs. Be sure your answer shows the payoffs at expiration and proves that these payoffs are riskless.
17. Suppose the U.S. interest rate for the next six months is 1.5 percent (annual compounding). The foreign interest rate is 2 percent (annual compounding). The spot price of the foreign currency in dollars is $1.665. The forward price is $1.664. Determine the correct forward price and recommend an arbitrage strategy.
18. (Concept Problem) Suppose that there is a futures contract on a portfolio of stocks that currently are worth $100. The futures has a life of 90 days, and during that time the stocks will pay dividends of $0.75 in 30 days, $0.85 in 60 days, and $0.90 in 90 days. The simple interest rate is 12 percent.
    a. Find the price of the futures contract assuming that no arbitrage opportunities are present.
    b. Find the value of θ, the cost of carry in dollars.
19. Identify and provide a brief explanation of the factors that affect the spot price of a storable asset.

20. If futures prices are less than spot prices, the explanation usually given is the convenience yield. Explain what the convenience yield is. Then identify certain assets on which convenience yields are more likely to exist and other assets on which they are not likely to be found.
21. Explain why American call options on futures could be exercised early when call options on the spot are not. Assume that there are no dividends.
22. Describe two problems in using the Black option on futures pricing model for pricing options on Eurodollar futures.
23. What is a contango market? How do we interpret the cost of carry in a contango market? What is a backwardation market? How do we explain the cost of carry in a backwardation market?
24. On September 26 the spot price of wheat was \$3.5225 per bushel and the price of a December wheat futures was \$3.64 per bushel. How do you interpret the futures price if there is no risk premium in the futures market?
25. Explain why the Black option on futures pricing model is simply a pricing model for options on instruments with a zero cost of carry.

10

FUTURES ARBITRAGE STRATEGIES

This chapter examines arbitrage futures trading strategies. We discussed arbitrage frequently in previous chapters. Arbitrage is one of the mechanisms that links derivative prices to spot prices. Without arbitrage, the markets would be far less efficient. Derivative and spot prices, however, do not always conform to their theoretical relationships. When this happens, arbitrageurs step in and execute profitable transactions that quickly drive prices back to their theoretical levels. This chapter illustrates some of the arbitrage transactions important to the proper functioning and efficiency of the futures markets. It should be noted that while we sometimes ignore transaction costs in these examples, in practice any arbitrage opportunities must be evaluated by considering whether the profits cover the transaction costs.

Our approach here is to examine four groups of contracts: short-term interest rate futures contracts, intermediate- and long-term interest rate futures contracts, stock index futures contracts, and foreign exchange futures contracts. Within each group of contracts, we shall examine the unique challenges of arbitrage trading with these instruments.

SHORT-TERM INTEREST RATE ARBITRAGE

In the category of short-term interest rate futures, we shall look at Federal funds and Eurodollar futures strategies. But before we start looking at specific strategies, let us examine some basic concepts that are used in understanding the arbitrage transactions that lead to the prices of these types of futures contracts. We begin by looking at the notion of carry arbitrage and a related concept, the implied repo rate.

Carry Arbitrage and the Implied Repo Rate

In Chapter 9, we saw that the futures price is determined by the spot price and the cost of carry. The basic arbitrage transaction that determines this relationship often is referred to as a cash-and-carry arbitrage or the shortened name carry arbitrage. The investor purchases the security in the spot market and sells a futures contract. If the futures contract is held to expiration, the security's sale price is guaranteed.[1] Since the transaction is riskless, it should offer a return sufficient to cover the cost of carry or the financing charge from the purchase of the security assuming that the position is financed completely by borrowing. Because there is no risk, the investor will not earn a risk premium.

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[1]Recall that this is true because the spot price at expiration equals the futures price at expiration. The profit on the spot transaction is $S_T - S_0$ and the profit on the futures transaction is $-(f_T(T) - f_0(T))$. The total profit is $f_0(T) - S_0$. Thus, $f_0(T)$ is the effective sale price of the underlying asset.

Another way to approach this problem is to focus on the rate at which the security's purchase can be financed. Often the financing is obtained by means of a repurchase agreement. A repurchase agreement, or repo, is an arrangement with a financial institution in which the owner of a security sells that security to the financial institution with an agreement to buy it back, usually a day later. This transaction is referred to as an overnight repo. The repo thus is a form of a secured loan. The investor obtains the use of the funds to buy the security by pledging it as collateral. The interest charged on a repo is usually quoted and calculated as if there were 360 days in a year, but here we shall use the assumption of a full 365-day year.

Repurchase agreements are frequently used in transactions involving government securities. Overnight repos are more common, but longer-term arrangements, called term repos, of up to two weeks are sometimes employed. In cash-and-carry transactions, the security is considered as being financed by using a repo. If the return from the transaction is greater than the repo rate, the arbitrage will be profitable.

From Chapter 9, the futures-spot price relationship is

$$f_0(T) = S_0 + \theta.$$

Since we are focusing on securities here, there is no significant storage cost; thus, the cost of carry, θ, is strictly the interest, iS_0. Now let us define the implicit interest cost as

$$\theta = f_0(T) - S_0.$$

Thus, θ is the implied cost of financing expressed in dollars. Suppose we express it as a percentage of the spot price, θ/S_0, and denote this as $\hat{r}$. Then $\hat{r}$ is the implied repo rate. If the cost of financing the position—the actual repo rate—is less than the implied repo rate, the arbitrage will be profitable.

Readers familiar with the concept of internal rate of return (IRR) will find that it is analogous to the implied repo rate. If the arbitrage brings no profit, the cost of financing is the implied repo rate. In a capital budgeting problem, a zero net present value (NPV) defines the internal rate of return. If the opportunity cost is less than the IRR, the project is attractive and produces a positive NPV. Likewise, an arbitrage is profitable if it can be financed at a rate lower than the implied repo rate.

For example, suppose there is a security that matures at time T and an otherwise identical security that matures at an earlier time, t. There is also a futures contract that expires at time t. You buy the longer-term security at a price of S_0, finance it at the rate r (an annualized rate), and sell a futures contract. At time t, the futures expires and you deliver the security. You have effectively sold the security at t for a price of $f_0(t)$. The profit from the transaction is

$$\Pi = f_0(t) - S_0(1 + r)^t.$$

The term $S_0(1 + r)^t$ reflects what you paid for the security, factored up by the cost of financing over period t. The implied repo rate, $\hat{r}$, is the cost of financing that produces no arbitrage profit; therefore, solving for the rate, we have

$$\hat{r} = \left(\frac{f_0(t)}{S_0}\right)^{1/t} - t.$$

The exponent 1/t can be easily interpreted as 365/days, where "days" is the number of days that the transaction is held in place. Raising $f_0(t)/S_0$ to this power just annualizes the rate.

There is yet another approach to understanding the basic carry arbitrage. Suppose we buy the security that matures at time T and simultaneously sell the futures contract that expires at time t ($t < T$). When the futures contract expires, we deliver the security, which has a remaining maturity of $T - t$. The net effect is that we have taken a security that matures at T and shortened its maturity to t. Thus, we have created a synthetic t-period instrument. If the return from the synthetic t-period instrument is greater than the return from an actual security maturing at t, prices are out of line and an arbitrage profit is possible.

Federal Funds Futures Carry Arbitrage and the Implied Repo Rate

Banks keep reserves at Federal Reserve Banks to fulfill their capital reserve requirements. Reserve balances in excess of reserve requirements held by one bank can be loaned to other banks with reserve balance deficiencies. These loans are typically very short term, usually overnight. Federal funds (also known as Fed funds) are reserve balances held at Federal Reserve Banks that can be loaned to other banks. Federal funds can be held by commercial banks, thrift institutions, Federal agencies, branches of foreign banks in the U.S., and government securities dealers. Federal funds rates are highly correlated with Treasury bill rates, LIBOR, commercial paper rates and somewhat correlated with longer term corporate and treasury rates. For example, the correlation coefficient between overnight effective Federal funds rate and one-month LIBOR is estimated to be more than 0.99. Hence arbitrageurs can use other interest rate instruments in their trading activities rather than just Federal funds. Federal funds are quoted on an add-on interest basis using an actual day count and a 360-day year.

The federal funds futures contract covers $5,000,000 of federal funds. Hence, one basis point is worth $41.67 (= $5,000,000 × 0.0001 × (30/360)). Prices are quoted as 100 minus the average daily Federal funds overnight rate for the delivery month. If the average daily Federal funds overnight rate is 5 percent, then the futures contract is quoted as 95.00. If a trader believes that short-term interest rates will rise, she could sell Federal funds futures contracts.

We shall use Federal funds futures contracts to illustrate a potential arbitrage strategy. The following strategy is based on the assumption that LIBOR and Federal funds rate are perfect substitutes; that is, there is no basis risk. We shall say more about this assumption after the illustration. Suppose we shall lend based on the two-month LIBOR rate and sell a Fed funds futures contract expiring in one month. This transaction creates a synthetic one-month LIBOR-based loan, with a rate that should equal the rate on an actual one-month LIBOR rate. If it does not, arbitrageurs will be attracted to the strategy and their transactions will drive down the two-month LIBOR rate and drive down the Fed funds futures price, or vice versa, until the synthetic LIBOR-based loan and the actual LIBOR have the same rates.

An Example Table 10.1 illustrates the carry arbitrage transaction with Federal funds futures contracts and LIBOR. This is an example of a situation in which lending based on longer-term LIBOR, the selling of a Fed funds futures contract that expires before the LIBOR loan matures, and borrowing based on the Fed funds rate when the futures expires creates a synthetic shorter-term LIBOR loan. The return on this synthetic LIBOR loan is about 23 basis points higher than the return on an actual LIBOR loan with the same maturity.

There are, however, several limitations to the effectiveness of this strategy. The effectiveness of this strategy hinges critically on the assumption of no basis risk between one-month LIBOR and Fed funds rates. Although the Fed funds rate is a rather volatile interest rate, historically, the difference between Fed funds on a short-term LIBOR rates is only a few basis points. Recall historical data confirms that the correlation coefficient between Fed funds and one-month LIBOR is extremely high. This makes sense because banks decide where to lend their excess reserves either to other banks through the Federal Reserve Bank or to other banks through the inter-bank market (LIBOR). Federal funds futures contracts are cash settled based on the average daily effective rate observed during the delivery month. Hence within the delivery month, there are considerable differences between short-term LIBOR rates and the implied Federal funds futures rate.[2] Another limitation to the carry arbitrage is that the repo rate is not fixed for the full time period. As noted earlier, some term repos for up to two weeks are available, but most repo financing is overnight. In either case, the financing rate on this strategy is

[2]The arbitrage described here has other subtle risks. See Footnote 3 for an issue related to the Eurodollar futures contract that also applies to the Fed Funds futures contract.

Table 10.1 Federal Funds Carry Arbitrage

Scenario: On March 1, the one-month LIBOR rate is 5.50 percent and the two-month LIBOR rate is 6.00 percent. The April Fed funds futures is quoted at 93.75. An arbitrage opportunity is available. Contract size is $5,000,000 and this example illustrates the arbitrage strategy for one contract.

Date	Spot Market	Futures Market
March 1	Borrow present value of $5,000,000 at one-month LIBOR ($4,977,187.89); lend same amount at two-month LIBOR.	Sell one April Fed funds futures contract at 93.75.
April 1	Repay borrowing (–$5,000,000). Payoff present value of loan.	Buy one April Fed funds futures contract to offset initial futures position.

Analysis: The key insight is that the payoff required on April 1st of the two-month LIBOR loan depends on the one-month LIBOR rate observed on April 1st. Also, we assume no basis risk between one-month LIBOR rates and Fed funds futures implied rates as long as we are not in the delivery month. The loan balance due on May 1st is:

$$LB_2 = LB_0\left(1 + r_2\left(\frac{60}{360}\right)\right) = \$4{,}977{,}187.89\left(1 + 0.06\left(\frac{60}{360}\right)\right) = \$5{,}026{,}959.77$$

Assuming the one-month LIBOR rate observed on April 1st is 7.25 percent, then the present value of the initial two-month loan after one month is:

$$PV_1(LB_2) = \frac{LB_2}{\left(1 + r_{1,1}\left(\frac{30}{360}\right)\right)} = \frac{\$5{,}026{,}959.77}{\left(1 + 0.0725\left(\frac{30}{360}\right)\right)} = \$4{,}996{,}770.94.$$

Assuming no basis risk, the Fed funds futures price in one month is 92.75 (=100 – 7.25). Therefore, the gain on the Fed funds futures contract is $4,166.67 (=(93.75 – 92.75) × ($5,000,000) × (30/360)/100). Adding the gain on the futures contract with the proceeds from the present value of the two-month loan, we have $5,000,937.61. Paying off the one-month loan provides arbitrage profits of $937.61. The implied one-month repo rate, therefore, is

$$\left(\frac{\$5{,}000{,}937.61}{\$4{,}977{,}187.89} - 1\right)\frac{360}{30} = 0.0573,$$

or 5.73 percent. The one-month LIBOR rate is 5.50 percent, indicating the potential arbitrage opportunity.

unknown when the arbitrage is executed. Each time a repo matures, new financing must be arranged, and the rate may well be much higher than originally expected. A number of other limitations, such as transaction costs, come into play when determining the implied repo rate from other futures instruments. There are several other very technical limitations that are beyond the scope of this introductory book.

Eurodollar Arbitrage

The Eurodollar deposit and Eurodollar derivatives markets are large markets in which foreign banks and foreign branches of U.S. banks issue dollar-denominated deposits and often commit to forward and futures transactions. Although the Eurodollar market is highly efficient, there may be occasional arbitrage opportunities resulting from violations of the carry arbitrage relationship between spot and futures or forward prices.

An Example Table 10.2 illustrates a situation in which a London bank, needing to issue a 180-day Eurodollar time deposit, finds that it can get a better rate by issuing a 90-day time deposit and selling a futures expiring in 90 days. By stringing the 90-day time deposit to the futures contract, the bank creates a synthetic 180-day time deposit with a better rate.

It might appear that the result was contingent on the rate on the time deposit issued on December 16—a rate that was not known back in September. In fact, the result of this transaction was known when it was executed. The 90-day time deposit was issued at a rate of 0.0825(90/360) = 0.020625 for the 90 days. The Eurodollar futures was sold at a rate of 0.0863(90/360) = 0.021575. Thus, if the bank issued a 90-day time deposit at 0.020625 and followed that with a 90-day time deposit at 0.021575, the overall rate for 180 days would be

$$(1.020625)(1.021575) - 1 = 0.042645.$$

Annualizing this rate gives

$$(1.042645)^{(365/180)} - 1 = 0.0884,$$

which is the rate obtained by the bank.[3]

Table 10.2 Eurodollar Arbitrage

Scenario: The rates available in the spot and futures markets are such that the bank can obtain a better rate by doing the following.

Date	Spot Market	Futures Market
September 16	The 90-day time deposit rate is 8.25.	December Eurodollar IMM Index is at 91.37.
		Price per $100 face value:
		100 – (100 – 91.37)(90/360) = 97.8425.
	Issue 90-day time deposit	Price per contract: $978,425.
	for $10,000,000	**Sell 10 contracts**
December 16	The 90-day time deposit matures	December Eurodollar IMM Index is at 92.04.
	and the bank owes $10,000,000	Price per $100 face value:
	(1 + 0.0825(90/360)) = $10,206,250.	100 – (100 – 92.04)(90/360) = 98.01.
	The rate on new 90-day time deposits is 7.96.	Price per contract: $980,100.
	Issue new 90-day time deposit	**Buy 10 contracts**
	for $10,223,000	

Analysis: On September 16, the bank received $10,000,000 from the newly issued 90-day time deposit. On December 16, it bought back its ten futures contracts at a loss of 10($980,100 – $978,425) = $16,750. It issued $10,223,000 of new time deposits, using $16,750 to cover the loss in the futures account and the remaining $10,206,250 to pay off the maturing 90-day time deposit. On March 16 it paid off the new time deposit, owing

$$\$10{,}223{,}000[1 + 0.0769(90/360)] = \$10{,}426{,}438.$$

Thus, on September 16, it received $10,000,000 and on March 16, it paid out $10,426,438. It had no other cash flows in the interim. Thus, its effective borrowing cost for 180 days was

$$\left(\frac{\$10{,}426{,}438}{\$10{,}000{,}000}\right)^{365/180} - 1 = 0.0884,$$

which is 23 basis points less than the cost of a 180-day Eurodollar time deposit.

[3]The arbitrage described here is not a perfect transaction in that the outcome is not without a small amount of risk. Because the Eurodollar futures contract settles at expiration as a discount instrument (that is, by subtracting rate*days/360 from par) and spot Eurodollars are priced as add-on instruments (that is, by multiplying par by 1 + rate*days/360), the spot and futures prices do not precisely converge. This characteristic of the Eurodollar market will prevent the arbitrage from being risk-free. Arbitrageurs would consider the potential for small variations in the outcome that may or may not be material to the overall success of the strategy.

INTERMEDIATE- AND LONG-TERM INTEREST RATE ARBITRAGE

Intermediate- and long-term interest rate futures include Treasury note and Treasury bond futures. As we noted in earlier chapters, these instruments are virtually identical. Here we shall concentrate on the Treasury bond contract. Recall that the contract is based on the assumption that the underlying bond has a 6 percent coupon rate and a maturity or call date of not less than 15 years.

Suppose you are short the March 2006 contract. As the holder of the short position, you have the choice of which day during the delivery month to make delivery and which bond from among the eligible bonds you will deliver. Let us assume you have decided to deliver the bonds with a coupon of 5 1/2 percent, and the bonds mature on August 15, 2028. Delivery will be made on Friday, March 7, 2006.

Since the contract assumes delivery of a bond with a 6 percent coupon, the delivery of the 5 1/2 requires an adjustment to the price paid by the long to the short. The adjustment is based on the CBOT's conversion factor system. The conversion factor, CF, is defined for each eligible bond for a given contract. The CF is the price of a bond with a face value of $1, coupon and maturity equal to that of the deliverable bond, and yield of 6 percent. The maturity is defined as the maturity of the bond on the first day of the delivery month. If the bond is callable, the call date is substituted for the maturity date. The CF for the 5 1/2 of 2028 would be the price of a bond with a face value of $1; coupon of 5 1/2 percent; maturity equal to the time remaining from March 1, 2006, the first day of the month, to August 15, 2028, the maturity date of the bond; and yield of 6 percent. That same bond delivered on the June 2006 contract would have a different CF because it would have a different maturity—June 2006 rather than March 2006. A different bond delivered on the March 2006 contract would have a different conversion factor.

The conversion factor system is designed to place all bonds on an equivalent basis for delivery purposes, which is designed to reduce the possibility that a single bond will be in excessive demand for delivery. If the holder of the short position delivers a bond with a coupon greater than 6 percent, the CF will be greater than 1. The short will then receive more than the futures price in payment for the bond. If the coupon is less than 6 percent, the CF will be less than 1 and the short will receive less than the futures price in payment for the bond.

Tables of conversion factors are available, and there is a specific formula for determining the conversion factor, which is provided in Appendix 10. Your software includes the Excel spreadsheet CF7e.xls that calculates the conversion factor, and both the spreadsheet and the conversion factor are explained in the appendix. In this problem, the CF for the 5 1/2 of 2028 delivered on the March 2006 contract would be 0.9389. To determine the invoice price—the amount the long pays to the short for the bond—multiply the CF by the settlement price on the position day. Then the accrued interest from the last coupon payment date until the delivery date is added:[4]

Invoice price = (Settlement price on position day) (Conversion factor) + Accrued interest.

In this problem, assume the settlement price on Wednesday, March 8, which is called the position day, was 112, or $112,000. The bond has coupon payment dates of February 15 and August 15. Thus, the last

[4]The accrued interest is the amount of interest that has built up since the last coupon date. Most bonds pay interest semiannually on the maturity day and six months hence. Thus a bond maturing on February 15, 2021, would pay interest every year on February 15 and August 15. The accrued interest is the semiannual coupon times the number of days since the last coupon date divided by the number of days between the last coupon date and the next coupon date. It is simply a proration of the next coupon. The buyer of a bond will receive the full next coupon, but, because she is not entitled to all of it, must pay the seller the accrued interest. The actual correct bond price will accurately include the accrued interest in it, though the quoted price will not.

coupon payment date was February 15, 2006. The number of days from February 15 to March 7 is 20, and the number of days between coupon payment dates of February 15, 2006, and August 15, 2006, is 181. Thus, the accrued interest is

$$\$100{,}000(0.055/2)(20/181) = \$303.87.$$

The invoice price therefore is

$$\$112{,}000(0.9389) + \$303.87 = \$105{,}460.67.$$

On the next day, called the notice of intention day, Thursday, March 9, the holder of the long position receives an invoice of $105,460.67 and must pay this amount and accept the bond on the delivery day, Friday, March 10.

Table 10.3 presents CFs and invoice prices for other bonds deliverable on the March 2006 contract. Note how the conversion factors vary directly with the level of the coupon.

Table 10.3 Conversion Factors and Invoice Prices for Deliverable Bonds (March 2006 T-Bond Futures Contract)

Contract price: 112 ($112,000)
Delivery date: March 10, 2006

Coupon	Maturity Date	CF	Accrued Interest	Invoice Price
8.125%	May 15, 2021	1.2083	6,602	141,926
8.125%	August 15, 2021	1.2102	4,570	140,118
8.000%	November 15, 2021	1.2000	6,500	140,900
7.250%	August 15, 2022	1.1285	4,078	130,468
7.625%	November 15, 2022	1.1687	6,195	137,092
7.125%	February 15, 2023	1.1177	4,008	129,192
6.250%	August 15, 2023	1.0265	3,516	118,486
7.500%	November 15, 2024	1.1663	6,094	136,714
7.625%	February 25, 2025	1.1813	4,082	136,387
6.875%	August 15, 2025	1.0990	3,867	126,952
6.000%	February 15, 2026	1.0000	3,375	115,375
6.750%	August 15, 2026	1.0871	3,797	125,554
6.250%	November 15, 2026	1.0293	5,078	120,356
6.625%	February 15, 2027	1.0735	3,727	123,958
6.375%	August 15, 2027	1.0446	3,586	120,580
6.125%	November 15, 2027	1.0150	4,977	118,655
5.500%	August 15, 2028	0.9389	3,094	108,254
5.250%	November 15, 2028	0.9081	4,266	105,968
5.250%	February 15, 2029	0.9075	2,953	104,590
6.125%	August 15, 2029	1.0154	3,445	117,176
6.250%	May 15, 2030	1.0316	5,078	120,615
5.375%	February 15, 2031	0.9198	3,023	106,047

Note: These calculations were done on a computer and can vary from the amounts obtained if done by hand.

Determining the Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract

As previously explained, the specifications on the Treasury bond contract allow delivery of many different bonds, subject to the minimum 15-year maturity or call date. As we noted, the conversion factor system is designed to place all bonds on an equivalent basis for delivery purposes. If the conversion factor system were perfect, all bonds would be equally desirable for delivery. But the conversion factor system is not perfect and all bonds are not equally desirable for delivery. At any given time prior to expiration, it is impossible to determine which bond will be delivered. It is, however, possible to identify the bond that is most likely to be delivered. That bond is referred to as the cheapest-to-deliver or CTD.

Suppose it is December 2, 2005, and you are interested in determining the bond that is most likely to be delivered on the upcoming March contract. The procedure involves a series of calculations that we shall now illustrate for one particular bond—the 6 1/4s that mature on May 15, 2030.

If the holder of a long position in this bond also holds a short position in the T-bond futures contract, that trader can elect to maintain the position until expiration and deliver this particular bond. Even if another bond would be cheaper to deliver, the trader always has the option to deliver the bond already held. The cost of delivering the particular bond is the net profit or loss from buying the bond, selling a futures, holding the position until expiration, and then delivering the bond. Thus, the trader incurs the cost of carry on the bond held. Remember that this cost is somewhat offset by the coupons received on the bond.

For evaluating at time t the best bond to deliver at time T, the general expression for the cost of delivering a bond is

$$f_0(T)(CF) + AI_T - [(B + AI_t)(1 + r)^{(T-t)} - CI_{t,T}],$$

where AI_T is the accrued interest on the bond at T, the delivery date; AI_t is the accrued interest on the bond at t, today; r is the risk-free rate that represents the interest lost on the funds invested in the bond; and $CI_{t,T}$ is the value at time T of the coupons received and reinvested over the period of t to T. The term inside the brackets is the spot price of the bond (quoted price plus accrued interest) factored up by the cost of carry and reduced by the compound future value of any coupons received while the position is held. These coupons, of course, help offset the cost of carry, and by subtracting them we are simply reflecting the net cost of carry. The bracketed term is, thus, the forward price of the bond. The first two terms are the amount the trader would receive from delivering the bond. This is the invoice price.

The futures price is 112, and the conversion factor for our bond is 1.0316. The accrued interest on December 2 is 0.29, and the accrued interest on March 7, the day we shall assume delivery, is 1.93. The price quoted for the bond is 121. The reinvestment rate is 4.0 percent. There are 95 days between December 2 and March 7. The invoice price for the futures is

$$112(1.0316) + 1.93 = 117.47.$$

There are no interim coupons paid. Thus, the forward price of the bond is

$$(121 + 0.29)(1.04)^{(95/365)} = 122.53.$$

Hence, the bond would cost 5.06 (= 122.53 − 117.47) more than it would return.

This conclusion by itself does not enable us to make a decision. We can only compare this figure for one bond to that for another. Let us consider a second bond, the 7 1/4s maturing on August 15, 2022. Its conversion factor is 1.1285, and its price is 127 24/32 or 127.75. The accrued interest is 2.15 on December 2 and 0.40 on March 7. The coupon of 3.625 received on February 15 is reinvested at 4.0 percent for 20 days and grows to a value of

$$3.625(1.04)^{(20/365)} = 3.63.$$

Thus, the forward price of the bond is

$$(127.75 + 2.15)(1.04)^{(95/365)} - 3.63 = 127.60.$$

The invoice price is

$$112(1.1285) + 0.40 = 126.79.$$

Thus, the difference between the amount received and the amount paid is $126.79 - 127.60 = -0.81$.

Therefore, it is clear that the 7 1/4 percent bond is better to deliver than the 6 1/4 percent bond. Of course, this calculation should be done for all bonds that are eligible for delivery. The bond for which the difference between the amount received and the amount paid is the maximum is the cheapest bond to deliver.

Table 10.4 Cheapest-to-Deliver Bond on the Treasury Bond Futures Contract

Current date:	December 2, 2005
Delivery date:	March 7, 2006
Reinvestment rate:	4%
Futures price:	112

Coupon	Maturity	Ask Price	Forward Price	Invoice Price	Difference	Accrued Interest at t	Accrued Interest at T	Future Value of Coupon	CF
8.125%	May 15, 2021	136 1/16	137.8440	137.8383	−0.0057	0.3816	2.5138	0.0000	1.2083
8.125%	August 15, 2021	136 5/16	136.0712	135.9964	−0.0748	2.4066	0.4489	4.0712	1.2102
8.000%	November 15, 2021	135 5/16	137.0804	136.8756	−0.2048	0.3757	2.4751	0.0000	1.2000
7.250%	August 15, 2022	127 3/4	127.5974	126.7906	−0.8068	2.1474	0.4006	3.6328	1.1285
7.625%	November 15, 2022	132 5/16	134.0318	133.2560	−0.7758	0.3581	2.3591	0.0000	1.1687
7.125%	February 15, 2023	126 25/32	126.6440	125.5775	−1.0665	2.1104	0.3936	3.5702	1.1177
6.250%	August 15, 2023	116 7/8	116.8127	115.3159	−1.4968	1.8512	0.3453	3.1317	1.0265
7.500%	November 15, 2024	133 1/16	134.7836	132.9409	−1.8427	0.3522	2.3204	0.0000	1.1663
7.625%	February 25, 2025	134 27/32	134.4830	132.5161	−1.9669	2.0513	0.2106	3.8166	1.1813
6.875%	August 15, 2025	125 7/8	125.7789	123.4648	−2.3140	2.0363	0.3798	3.4449	1.0990
6.000%	February 15, 2026	115 1/8	115.0952	112.3315	−2.7637	1.7772	0.3315	3.0065	1.0000
6.750%	August 15, 2026	125 1/16	124.9833	122.1301	−2.8531	1.9993	0.3729	3.3823	1.0871
6.250%	November 15, 2026	122	123.5483	117.2114	−6.3369	0.2935	1.9337	0.0000	1.0293
6.625%	February 15, 2027	123 13/16	123.7457	120.5976	−3.1481	1.9623	0.3660	3.3196	1.0735
6.375%	August 15, 2027	120 25/32	120.7338	117.3460	−3.3878	1.8882	0.3522	3.1944	1.0446
6.125%	November 15, 2027	117 19/32	119.0909	115.5738	−3.5171	0.2876	1.8950	0.0000	1.0150
5.500%	August 15, 2028	109 7/16	109.4502	105.4640	−3.9863	1.6291	0.3039	2.7559	0.9389
5.250%	November 15, 2028	106 1/8	107.4630	103.3265	−4.1365	0.2465	1.6243	0.0000	0.9081
5.250%	February 15, 2029	106 5/32	106.1858	101.9270	−4.2588	1.5550	0.2901	2.6306	0.9075
6.125%	August 15, 2029	118 5/8	118.6059	114.0688	−4.5371	1.8142	0.3384	3.0691	1.0154
6.250%	May 15, 2030	120 29/32	122.4433	117.4710	−4.9723	0.2935	1.9337	0.0000	1.0316
5.375%	February 15, 2031	125 11/32	125.5449	103.3200	−22.2249	1.5921	0.2970	2.6933	0.9198

Note: These calculations were done on a computer and can vary from the amounts obtained if done by hand.

Table 10.4 presents these calculations for the March 2006 contract evaluated on December 2. Twenty-two eligible bonds are shown. The column labeled "Difference" is the difference between the invoice price and the forward price. All of these values are negative as they should be; otherwise one could earn an easy arbitrage profit. The cheapest bond to deliver is the one in which Difference is the closest to zero. In this case it is the 8 1/8 of May 15, 2021. The Excel spreadsheet CTD7e.xls will automatically do these calculations for you. Software Demonstration 10.1 illustrates how to use CTD7e.xls.

The cheapest bond to deliver is important for several reasons. Any futures contract must reflect the behavior of the spot price. In the case of Treasury bond and note futures, the so-called "spot price" is not easy to determine. The cheapest bond to deliver is the bond that represents the spot instrument that the futures contract is tracking. Therefore, the cost of carry model examined in Chapter 9 would apply only to the cheapest bond to deliver.

Delivery Options

The characteristics of the Treasury bond futures contract create some interesting opportunities for alert investors. Specifically, the contract contains several imbedded options. While these delivery options are not formally traded in the same way stock options are, they have many of the characteristics of the options we studied in earlier chapters. We shall examine some of these options here.

Wild Card Option The wild card option results from a difference in the closing times of the spot and futures markets. The Treasury bond futures contract stops trading at 3:00 P.M. Eastern time. The spot market for Treasury bonds operates until 5:00 P.M. Eastern time. During the delivery month, the holder of a short position knows the settlement price for that day at 3:00 P.M. Multiplying the settlement price by the conversion factor gives the invoice price the holder would receive if a given bond were delivered. This figure is locked in until the next day's trading starts.

During the two-hour period after the futures market closes, the spot market continues to trade. If the spot price declines during those two hours, the holder of a short futures position may find it attractive to buy a bond and deliver it. Because the futures market is closed and the invoice price is fixed, the futures market is unable to react to the new information that drove the spot price down. Moreover, the short has until about 9:00 P.M. to make the decision to deliver.

Let us use the following symbols:

f_3 = futures price at 3:00 PM

B_3 = spot price at 3:00 PM

CF = conversion factor of bond under consideration

You hold a short position in the futures contract that is expiring during the current month. Assume you own 1/CF bonds. Why this unusual amount? If you do not own any bonds, your position will be quite risky. Also, the bond under consideration should be the cheapest bond to deliver. That way your risk is quite low. Unexpected changes in the futures price will be approximately matched by changes in the value of the 1/CF bonds. We require the case that CF > 1.0.

If you make delivery that day, you will be required to deliver one bond per contract. You own only 1/CF bonds, so you will have to buy 1 − 1/CF additional bonds. If the bonds' spot price declines sufficiently between 3:00 P.M. and 5:00 P.M., you may be able to buy the additional bonds at a price low enough to make a profit. These additional bonds are called the *tail.*

Now suppose that at 5:00 P.M. the bond price is B_5. If you buy the additional 1 − 1/CF bonds at the price of B_5, and deliver them, your profit will be

$$\Pi = f_3(CF) - \left[\left(\frac{1}{CF}\right)B_5 + \left(1 - \frac{1}{CF}\right)B_5\right] = f_3(CF) - B_5.$$

The first term, $f_3(CF)$, is the invoice price. This is simply the 3:00 P.M. settlement price on the futures contract times the conversion factor on the bond. The invoice price is the amount you receive upon delivery. The terms in brackets denote values of the bonds you are delivering. The first term, $(1/CF)B_5$, is the 5:00 P.M. value of the 1/CF bonds you already owned. The second term is the cost of the 1 − 1/CF bonds bought at the 5:00 P.M. price. As indicated above, the expression simplifies to $f_3(CF) - B_5$. Note that the 3:00 P.M. bond price does not enter into the decision to deliver because the delivery decision is made at 5:00 P.M. By that time, the 1/CF bonds are worth $(1/CF)B_5$ and can be sold for that amount.

If the transaction is profitable, $\Pi > 0$. This requires that

$$B_5 < f_3(CF).$$

A trader can observe the spot price at 5:00 P.M. If the price is sufficiently low, the trader should buy the remaining 1 − 1/CF bonds and make delivery. The wild card option thus will be profitable if the spot price at 5:00 P.M. falls below the invoice price established at 3:00 P.M.

As an example, suppose that on March 2 the March futures contract has a settlement price at 3:00 P.M. of 101.8125. The cheapest bonds to deliver were the 12 1/2s maturing in about 22 years, which have a conversion factor of 1.464. We do not have the 3:00 P.M. spot price, but let us assume it is 149.65. Suppose we are short 100 contracts, which obligates us to deliver bonds with a face value of 100($100,000) = $10,000,000. Assume each bond has a face value of $1,000. Thus, we will have to deliver 10,000 bonds. To begin this strategy, we must have a position in the spot T-bonds; otherwise, our risk will be quite high. We weight that position by the conversion factor; that is, we hold bonds with a face value of $10,000,000(1/1.464) = $6,830,601—in other words, about 6,831 bonds. To make delivery, we will need to buy 3,169 bonds. For us to make a profit, the 5:00 P.M. price must decline to

$$B_5 < 101.8125(1.464) = 149.05,$$

or less. Thus, if the spot price declined by at least 0.60 by 5:00 P.M., it would pay to buy the remaining 3,169 bonds and make delivery.

The alternative to delivery is holding the position until the next day. In fact, that will always be the better choice if the conversion factor is less than 1.0. Note that if CF < 1.0, the hedged position will include 1/CF bonds, which exceeds 1.0. Thus, the hedger will hold more bonds than futures and there will be no tail, or additional bonds, to purchase at the lower 5:00 P.M. price. In that case, the wild card option is worthless. Moreover, if the price does not fall sufficiently by 5:00 P.M., holding the position is preferred over delivery. Of course, it is possible that the wild card option will never be worth exercising, meaning simply that it ends up, like many other options, out-of-the-money.

Quality Option The holder of the short position has the right to deliver any of a number of acceptable bonds. Sometimes the holder of the short position will be holding a bond that is not the best to deliver. A profit is sometimes possible by switching to another bond. This is called the quality option, because the deliverable bonds are considered to be of different quality; it is also sometimes called the switching option.

The value of this option arises because of changes in the term structure of interest rates. You may be holding a hedged position in Treasury bonds and futures, anticipating that you will make delivery of the bond that you hold on the futures contract. Then if the term structure changes, another bond may become the cheapest to deliver. Although you could always deliver the bond you hold, the right to switch to a more favorably priced bond has value.

The quality option exists, not only in Treasury bond and note markets, but also in the markets for many commodity futures. For example, the Chicago Board of Trade's wheat futures contract specifies that the holder of the short position can deliver any of four different grades of wheat. Other agricultural contracts have similar options. In fact, agricultural contracts usually grant the right to deliver at one of several locations. This feature, called the location option, has essentially the same economic effect as the quality option.

The quality option conveys a right to the holder of the short position. Since this right has value, the futures price will tend to be lower by the value of this option. In other words, the seller of a futures receives a lower price that reflects the valuable option attached to the contract. Likewise, the buyer pays a lower price for granting the seller this valuable right.

End-of-the-Month Option The last day for trading a T-bond futures contract is the eighth-to-last business day of the delivery month. Delivery can take place during the remaining business days. The invoice price during those final delivery days is based on the settlement price on the last trading day. Thus, during the last seven delivery days, the holder of the short position has full knowledge of the price that would be received for delivery of the bonds. This gives the holder of the short position the opportunity to watch the spot market for a fall in bond prices. The trader can continue to wait for spot prices to fall until the second-to-last business day, because delivery must occur by the last business day.

The end-of-the-month option, thus, is similar to the wild card option. There is a period during which spot prices can change while the delivery price is fixed. It is also related to the quality option, for the holder of the short position can also switch to another bond.

Timing Option Since the short is often permitted to make delivery on any day during the delivery month, he holds another valuable option. Suppose the Treasury bond pays a coupon that exceeds the cost of financing the position, which is the repo rate. Then the short should hold the long bond-short futures position as long as possible because it pays more than it costs. If the repo rate exceeds the bond coupon, then the position costs more to hold than it yields; thus, an early delivery is advised. Ignoring all other delivery options, this timing option would suggest that all deliveries would occur early or late in the month. As we have already noted, however, there are many other options that are in effect and early delivery would preclude the right to take advantage of some of these other options.

Determining the values of these delivery options is very difficult. Most contracts contain more than one option and, as noted earlier, the Treasury bond futures contract contains several delivery options. Thus, it is difficult to isolate them and observe their separate effects. There have been numerous studies, which are reviewed in Chance and Hemler (1993). Most of them found that the value of delivery options is fairly small. That does not mean they should be dismissed as insignificant. To the holder of one or more of these options, the value may at times be quite large. For our purposes, however, we can safely conclude that the economic effects are minor in comparison to the more important factors that determine the futures price.

Implied Repo, Carry Arbitrage and Treasury Bond Futures

Earlier in this chapter, we examined the concept of the implied repo rate in the context of the carry arbitrage model for Federal funds futures. The concept is equally applicable to Treasury bond futures.

First, we must identify the cheapest bond to deliver. If we buy that bond today (time 0), we pay the spot price plus the accrued interest. The sale of a repo finances the bond's purchase. This means that we borrow the funds by selling the bond and agreeing to buy it back at a specified later date. We simultaneously sell a futures contract expiring at T. During the period of 0 to T, we may receive coupons, which would be reinvested at some appropriate rate and accrue to some value at T. Let us denote this value of reinvested coupons as $CI_{0,T}$. Thus, we hold the position from 0 to T, collecting and reinvesting the coupons. At expiration, we deliver the bond, and effectively receive $(CF)(f_0(T))$, the futures price times the conversion factor, plus accrued interest for it. In an efficient market, there should be no arbitrage profit. Therefore, the amount we receive for the bond must equal the amount we paid for it plus the cost of carry:

$$(CF)(f_0(T)) + AI_T + CI_{0,T} = (B_0 + AI_0)(1 + \hat{r})^T,$$

where B_0 is the bond price when it is bought, AI_T is the accrued interest on the bond at expiration, AI_0 is the accrued interest when the bond is bought, and $\hat{r}$ is the implied repo rate. The left-hand side of the formula is the amount we receive upon delivery. The right-hand side is the amount paid for the bond, $B_0 + AI_0$, factored up by the cost of financing over the holding period, 0 to T. Solving for $\hat{r}$,

$$\hat{r} = \left[\frac{(CE)(f_0(T)) + AI_T + CI_{0,T}}{B_0 + AI_0}\right]^{(1/T)} - 1.$$

SOFTWARE DEMONSTRATION 10.1

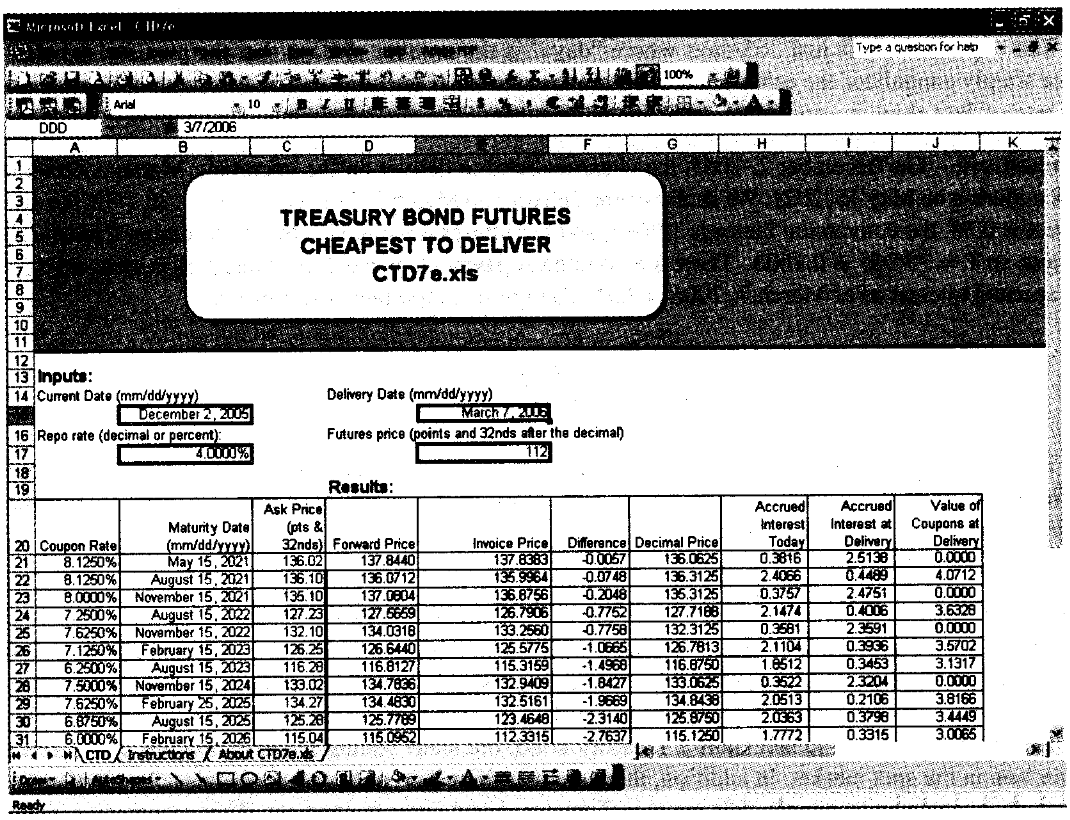
Identifying the Cheapest-to-Deliver Bond with the Excel Spreadsheet CTD7e.xls

The Excel spreadsheet CTD7e.xls is written in Excel 2002. It identifies the cheapest-to-deliver bond from among a set of bonds for the Chicago Board of Trade bond futures contract. To use the spreadsheet, you will need Windows 95 or higher. The spreadsheet is available as a download via the product support Web site. To access it:

1. Go to www.academic.cengage.com/aise.
2. Click on Instructor Resources or Student Resources.
3. Locate the title of this book on the Web page.
4. Download and install the spreadsheet using the link provided.

This spreadsheet is a read-only spreadsheet, meaning that it cannot be saved under the name CTD7e.xls, which preserves the original file.

Now let us work an example. Consider the problem in the text in which on December 2, 2005, you wish to identify the cheapest-to-deliver bond on the March 2006 contract. The current repo rate is 4.0 percent and the futures price is 112. You have identified 22 bonds that meet the 15-year maturity or callability requirement and are eligible for delivery.



Coupon Rate	Maturity Date (mm/dd/yyyy)	Ask Price (pts & 32nds)	Forward Price	Invoice Price	Difference	Decimal Price	Accrued Interest Today	Accrued Interest at Delivery	Value of Coupons at Delivery
8.1250%	May 15, 2021	136.02	137.8440	137.8383	-0.0057	136.0625	0.3816	2.5138	0.0000
8.1250%	August 15, 2021	136.10	136.0712	135.9964	-0.0748	136.3125	2.4066	0.4489	4.0712
8.0000%	November 15, 2021	135.10	137.0804	136.8756	-0.2048	135.3125	0.3757	2.4751	0.0000
7.2500%	August 15, 2022	127.23	127.5659	126.7906	-0.7752	127.7188	2.1474	0.4006	3.6328
7.6250%	November 15, 2022	132.10	134.0318	133.2560	-0.7758	132.3125	0.3581	2.3591	0.0000
7.1250%	February 15, 2023	126.25	126.6440	125.5775	-1.0665	126.7813	2.1104	0.3936	3.5702
6.2500%	August 15, 2023	116.28	116.8127	115.3159	-1.4968	116.8750	1.8512	0.3453	3.1317
7.5000%	November 15, 2024	133.02	134.7836	132.9409	-1.8427	133.0625	0.3522	2.3204	0.0000
7.6250%	February 25, 2025	134.27	134.4830	132.5161	-1.9669	134.8438	2.0513	0.2106	3.8166
6.8750%	August 15, 2025	125.28	125.7789	123.4648	-2.3140	125.8750	2.0363	0.3798	3.4449
6.0000%	February 15, 2026	115.04	115.0952	112.3315	-2.7637	115.1250	1.7772	0.3315	3.0065

Observe on the spreadsheet a section labeled **Inputs:**. Each cell that will accept inputs has a double-line border, and the values are in blue. Enter the current date in the form mm/dd/yyyy. Insert "12/2/2005" in the appropriate box. Then enter the delivery date "3/7/2006" in its box. The repo rate can be entered as either "4.0" or "0.04" in the appropriate box. The futures price should be entered as points and thirty-seconds. Thus, 112 3/32 should be entered as "112.03". The trailing zero can be omitted for values like "112.2" for 112 20/32. List the individual bonds in the section below. Consider the first bond. Let it have a coupon rate of 8.125 percent, a maturity date of May 15, 2021, and an ask price of 136 1/32. For the coupon rate enter "8.125" or "0.08125". Enter the maturity date as "5/15/2021". Enter the ask price the same way you did the futures price. Thus, 136 1/32 is entered as "136.01". Press F9 to recalculate, and the values will appear for each bond in the section labeled **Results:**. Output cells have a single-line border.

The calculations shown are the forward price; the invoice price; the difference between the invoice price and forward price; the decimal price, which is simply the ask price converted from thirty-seconds to decimals; the accrued interest on the current date; the accrued interest on the delivery date; and the value of the coupons at delivery. The last figure is the value of the coupons paid between the current date and the delivery date plus any interest paid on their reinvestment. To the right of each row are additional columns used to perform the calculations. These columns include the conversion factor and various other figures used in the calculations. The column labeled "Difference" is the value that you are looking for. The bond with the largest difference, which will be the one with the least negative value, is the cheapest bond to deliver.

The number of bonds you wish to analyze may differ. To accommodate additional bonds, copy all of the formulas from columns D through AE into new rows.

The exponent 1/T is just 365/days where "days" is the number of days that the position is held. This factor simply annualizes the calculation to obtain the rate $\hat{r}$. If the bond can be financed in the repo market at a rate of less than $\hat{r}$, profitable arbitrage is possible.

An Example On December 2, 2005, the cheapest bond to deliver on the upcoming March contract is the 8 1/8 maturing on May 15, 2021. We shall assume delivery on March 7. The spot price is 136 1/32, the accrued interest is 0.38, the conversion factor is 1.2083, and the futures price is 112. From December 2 to March 7 is 95 days, so $T = 95/365 = 0.2603$. There are no interest payments made during this period, hence, $CI_{0,T} = 0$. The accrued interest as of March 7, 2006 is 2.51. The implied repo rate is, therefore,

$$\hat{r} = \left[\frac{112\,(1.2083) + 2.51}{136.12695 + 0.38}\right]^{(1/0.2603)} - 1 = 0.038.$$

If the bond can be financed in the repo market for less than the implied repo rate, the arbitrage would be profitable. Note that the implied repo rate is very close to the actual repo rate, meaning that profitable arbitrage would not likely be possible.

Treasury Bond Futures Spreads and the Implied Repo Rate

In Chapter 7 we learned about option spreads, which are transactions that are long one option and short another. Spreads are also widely used in futures trading, especially in the Treasury bond contracts. Suppose a trader takes a long position in a futures contract. If this is the only transaction, the risk is quite high. One way to modify the risk is to sell short a Treasury bond, but short selling requires that the trader execute a transaction in the spot market. In addition, there are margin requirements on short sales. An alternative that is easy to execute is to simply sell another Treasury bond futures contract. That transaction could be executed within the same trading pit. In addition, the margin requirement on a spread is much lower than the margin requirement on either a long or a short position.

Suppose there is a futures contract expiring at time t and another expiring at time T with t coming before T. Suppose today (time 0) we sell the longer-term contract and buy the shorter-term contract. At time t, the